Chapter 2

Geometric Reasoning

2A Inductive and Deductive Reasoning

2-1 Using Inductive Reasoning to Make Conjectures
2-2 Conditional Statements
2-3 Using Deductive Reasoning to Verify Conjectures
Lab Solve Logic Puzzles
2-4 Biconditional Statements and Definitions

2B Mathematical Proof

2-5 Algebraic Proof
2-6 Geometric Proof
Lab Design Plans for Proofs
2-7 Flowchart and Paragraph Proofs
Ext Introduction to Symbolic Logic

A corn maze from the 7A Ranch near Hondo
**Vocabulary**

Match each term on the left with a definition on the right.

1. angle
   - A. a straight path that has no thickness and extends forever
2. line
   - B. a figure formed by two rays with a common endpoint
3. midpoint
   - C. a flat surface that has no thickness and extends forever
4. plane
   - D. a part of a line between two points
5. segment
   - E. names a location and has no size
   - F. a point that divides a segment into two congruent segments

**Angle Relationships**

Select the best description for each labeled angle pair.

6. linear pair or vertical angles
7. adjacent angles or vertical angles
8. supplementary angles or complementary angles

**Classify Real Numbers**

Tell if each number is a natural number, a whole number, an integer, or a rational number. Give all the names that apply.

9. 6
   - 9. natural number, whole number, integer
10. –0.8
    - 10. not a natural number, not a whole number
11. –3
    - 11. not a natural number, not a whole number, integer
12. 5.2
    - 12. not a natural number, not a whole number, rational number
13. \( \frac{3}{8} \)
    - 13. not a natural number, not a whole number, rational number
14. 0
    - 14. natural number, whole number, integer, rational number

**Points, Lines, and Planes**

Name each of the following.

15. a point
16. a line
17. a ray
18. a segment
19. a plane

**Solve One-Step Equations**

Solve.

20. \( 8 + x = 5 \)
21. \( 6y = -12 \)
22. \( 9 = 6s \)
23. \( p - 7 = 9 \)
24. \( \frac{z}{5} = 5 \)
25. \( 8.4 = -1.2r \)
### Key Vocabulary/Vocabulario

<table>
<thead>
<tr>
<th>English</th>
<th>Spanish</th>
</tr>
</thead>
<tbody>
<tr>
<td>conjecture</td>
<td>conjetura</td>
</tr>
<tr>
<td>counterexample</td>
<td>contraejemplo</td>
</tr>
<tr>
<td>deductive reasoning</td>
<td>razonamiento dedutivo</td>
</tr>
<tr>
<td>inductive reasoning</td>
<td>razonamiento inductivo</td>
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<tr>
<td>polygon</td>
<td>polígono</td>
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<tr>
<td>proof</td>
<td>demostración</td>
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<tr>
<td>quadrilateral</td>
<td>cuadrilátero</td>
</tr>
<tr>
<td>theorem</td>
<td>teorema</td>
</tr>
<tr>
<td>triangle</td>
<td>triángulo</td>
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</tbody>
</table>

### Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word **counterexample** is made up of two words: *counter* and *example*. In this case, *counter* means “against.” What is a counterexample to the statement “All numbers are positive”?

2. The root of the word **inductive** is *ducere*, which means “to lead.” When you are inducted into a club, you are “led into” membership. When you use inductive reasoning in math, you start with specific examples. What do you think inductive reasoning leads you to?

3. In Greek, the word *poly* means “many,” and the word *gon* means “angle.” How can you use these meanings to understand the term **polygon**?

### Geometry TEKS

<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>Geometry structure*</td>
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<tr>
<td>develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems</td>
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<td>make conjectures ... and determine the validity of the conjectures, ...</td>
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<tr>
<td>determine the validity of a conditional statement, its converse, inverse, and contrapositive</td>
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<tr>
<td>construct and justify statements about geometric figures and their properties</td>
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<tr>
<td>use logical reasoning to prove statements are true and find counterexamples to disprove statements that are false</td>
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<tr>
<td>use inductive reasoning to formulate a conjecture</td>
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<tr>
<td>use deductive reasoning to prove a statement</td>
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<tr>
<td>select an appropriate representation ... in order to solve problems</td>
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<tr>
<td>use numeric and geometric patterns to make generalizations about geometric properties ...</td>
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</tbody>
</table>

* Knowledge and skills are written out completely on pages TX28–TX35.
Reading Strategy: Read and Interpret a Diagram

A diagram is an informational tool. To correctly read a diagram, you must know what you can and cannot assume based on what you see in it.

<table>
<thead>
<tr>
<th>What You CAN Assume</th>
<th>What You CANNOT Assume</th>
</tr>
</thead>
<tbody>
<tr>
<td>✔ Collinear points</td>
<td>✗ Measures of segments</td>
</tr>
<tr>
<td>✔ Betweenness of points</td>
<td>✗ Measures of angles</td>
</tr>
<tr>
<td>✔ Coplanar points</td>
<td>✗ Congruent segments</td>
</tr>
<tr>
<td>✔ Straight angles and lines</td>
<td>✗ Congruent angles</td>
</tr>
<tr>
<td>✔ Adjacent angles</td>
<td>✗ Right angles</td>
</tr>
<tr>
<td>✔ Linear pairs of angles</td>
<td></td>
</tr>
<tr>
<td>✔ Vertical angles</td>
<td></td>
</tr>
</tbody>
</table>

If a diagram includes labeled information, such as an angle measure or a right angle mark, treat this information as given.

Try This

List what you can and cannot assume from each diagram.

1. 

2. 

Geometric Reasoning
Who uses this?
Biologists use inductive reasoning to develop theories about migration patterns.

Biologists studying the migration patterns of California gray whales developed two theories about the whales’ route across Monterey Bay. The whales either swam directly across the bay or followed the shoreline.

EXAMPLE
Identifying a Pattern
Find the next item in each pattern.

A
Monday, Wednesday, Friday, ...
Alternating days of the week make up the pattern.
The next day is Sunday.

B
3, 6, 9, 12, 15, ...
Multiples of 3 make up the pattern. The next multiple is 18.

C
←, ↖, ↑, ...
In this pattern, the figure rotates 45° clockwise each time.
The next figure is ↗.

1. Find the next item in the pattern 0.4, 0.04, 0.004, ...

When several examples form a pattern and you assume the pattern will continue, you are applying inductive reasoning. Inductive reasoning is the process of reasoning that a rule or statement is true because specific cases are true. You may use inductive reasoning to draw a conclusion from a pattern. A statement you believe to be true based on inductive reasoning is called a conjecture.

EXAMPLE
Making a Conjecture
Complete each conjecture.

A
The product of an even number and an odd number is ___ ? ___.
List some examples and look for a pattern.
(2)(3) = 6 (2)(5) = 10 (4)(3) = 12 (4)(5) = 20
The product of an even number and an odd number is even.
Complete each conjecture.

**B** The number of segments formed by \( n \) collinear points is __?__.

Draw a segment. Mark points on the segment, and count the number of individual segments formed. Be sure to include overlapping segments.

<table>
<thead>
<tr>
<th>Points</th>
<th>Segments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2 + 1 = 3</td>
</tr>
<tr>
<td>4</td>
<td>3 + 2 + 1 = 6</td>
</tr>
<tr>
<td>5</td>
<td>4 + 3 + 2 + 1 = 10</td>
</tr>
</tbody>
</table>

The number of segments formed by \( n \) collinear points is the sum of the whole numbers less than \( n \).

2. Complete the conjecture: The product of two odd numbers is __?__.

**Example 3** Biology Application

To learn about the migration behavior of California gray whales, biologists observed whales along two routes. For seven days they counted the numbers of whales seen along each route. Make a conjecture based on the data.

<table>
<thead>
<tr>
<th>Numbers of Whales Each Day</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Direct Route</strong></td>
</tr>
<tr>
<td>1 3 0 2 1 1 0</td>
</tr>
<tr>
<td><strong>Shore Route</strong></td>
</tr>
<tr>
<td>7 9 5 8 6 7</td>
</tr>
</tbody>
</table>

More whales were seen along the shore route each day. The data supports the conjecture that most California gray whales migrate along the shoreline.

3. Make a conjecture about the lengths of male and female whales based on the data.

<table>
<thead>
<tr>
<th>Average Whale Lengths</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length of Female (ft)</strong></td>
</tr>
<tr>
<td><strong>Length of Male (ft)</strong></td>
</tr>
</tbody>
</table>

To show that a conjecture is always true, you must prove it. To show that a conjecture is false, you have to find only one example in which the conjecture is not true. This case is called a counterexample. A counterexample can be a drawing, a statement, or a number.

**Inductive Reasoning**

1. Look for a pattern
2. Make a conjecture.
3. Prove the conjecture or find a counterexample.
Finding a Counterexample

Show that each conjecture is false by finding a counterexample.

A. For all positive numbers \( n, \frac{1}{n} \leq n \).
   Pick positive values for \( n \) and substitute them into the equation to see if the conjecture holds.
   - Let \( n = 1 \). Since \( \frac{1}{1} = 1 \) and \( 1 \leq 1 \), the conjecture holds.
   - Let \( n = 2 \). Since \( \frac{1}{2} = \frac{1}{2} \) and \( \frac{1}{2} \leq 2 \), the conjecture holds.
   - Let \( n = \frac{1}{2} \). Since \( \frac{1}{n} = \frac{1}{\frac{1}{2}} = 2 \) and \( 2 \neq \frac{1}{2} \), the conjecture is false.
   \( n = \frac{1}{2} \) is a counterexample.

B. For any three points in a plane, there are three different lines that contain two of the points.
   Draw three collinear points.
   If the three points are collinear, the conjecture is false.

C. The temperature in Abilene, Texas, never exceeds 100°F during the spring months (March, April, and May).

| Monthly High Temperatures (°F) in Abilene, Texas |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| 88  | 89  | 97  | 99  | 107 | 109 | 110 | 107 | 106 | 103 | 92  | 89  |

The temperature in May was 107°F; so the conjecture is false.

Show that each conjecture is false by finding a counterexample.

4a. For any real number \( x, x^2 \geq x \).
4b. Supplementary angles are adjacent.
4c. The radius of every planet in the solar system is less than 50,000 km.

<table>
<thead>
<tr>
<th>Planets’ Diameters (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
</tr>
<tr>
<td>4880</td>
</tr>
</tbody>
</table>

THINK AND DISCUSS

1. Can you prove a conjecture by giving one example in which the conjecture is true? Explain your reasoning.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, describe the steps of the inductive reasoning process.
**GUIDED PRACTICE**

1. **Vocabulary** Explain why a conjecture may be true or false.

2. Find the next item in each pattern.
   - March, May, July, ...
   - $\frac{1}{3}$, $\frac{2}{4}$, $\frac{3}{5}$, ...
   - $\text{ } \text{ } \text{ }$, $\text{ } \text{ } \text{ } \text{ }$, $\text{ } \text{ } \text{ } \text{ }$, ...

3. Complete each conjecture.
   - The product of two even numbers is __?__.
   - A rule in terms of $n$ for the sum of the first $n$ odd positive integers is __?__.

4. **Biology** A laboratory culture contains 150 bacteria. After twenty minutes, the culture contains 300 bacteria. After one hour, the culture contains 1200 bacteria. Make a conjecture about the rate at which the bacteria increases.

5. **Example** Show that each conjecture is false by finding a counterexample.
   - Kennedy is the youngest U.S. president to be inaugurated.
   - Three points on a plane always form a triangle.
   - For any real number $x$, if $x^2 \geq 1$, then $x \geq 1$.

**PRACTICE AND PROBLEM SOLVING**

1. Find the next item in each pattern.
   - 8 A.M., 11 A.M., 2 P.M., ...
   - 75, 64, 53, ...
   - $\text{ } \text{ } \text{ }$, $\text{ } \text{ } \text{ } \text{ }$, $\text{ } \text{ } \text{ } \text{ }$, ...

2. Complete each conjecture.
   - A rule in terms of $n$ for the sum of the first $n$ even positive integers is __?__.
   - The number of nonoverlapping segments formed by $n$ collinear points is __?__.

3. **Industrial Arts** About 5% of the students at Lubbock High School usually participate in the robotics competition. There are 526 students in the school this year. Make a conjecture about the number of students who will participate in the robotics competition this year.

4. Show that each conjecture is false by finding a counterexample.
   - If $1 - y > 0$, then $0 < y < 1$.
   - For any real number $x$, $x^3 \geq x^2$.
   - Every pair of supplementary angles includes one obtuse angle.

5. Make a conjecture about each pattern. Write the next two items.
   - 2, 4, 16, ...
   - $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ...
   - $\text{ } \text{ } \text{ }$, $\text{ } \text{ } \text{ } \text{ }$, $\text{ } \text{ } \text{ } \text{ }$, ...

6. Draw a square of dots. Make a conjecture about the number of dots needed to increase the size of the square from $n \times n$ to $(n + 1) \times (n + 1)$.
Determine if each conjecture is true. If not, write or draw a counterexample.

24. Points $X$, $Y$, and $Z$ are coplanar.

25. If $n$ is an integer, then $-n$ is positive.

26. In a triangle with one right angle, two of the sides are congruent.

27. If $BD$ bisects $\angle ABC$, then $m\angle ABD = m\angle CBD$.

28. **Estimation** The Westside High School band is selling coupon books to raise money for a trip. The table shows the amount of money raised for the first four days of the sale. If the pattern continues, estimate the amount of money raised during the sixth day.

<table>
<thead>
<tr>
<th>Day</th>
<th>Money Raised ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>146.25</td>
</tr>
<tr>
<td>2</td>
<td>195.75</td>
</tr>
<tr>
<td>3</td>
<td>246.25</td>
</tr>
<tr>
<td>4</td>
<td>295.50</td>
</tr>
</tbody>
</table>

29. Write each fraction in the pattern $\frac{1}{11}$, $\frac{2}{11}$, $\frac{3}{11}$, ... as a repeating decimal. Then write a description of the fraction pattern and the resulting decimal pattern.

30. **Math History** Remember that a prime number is a whole number greater than 1 that has exactly two factors, itself and 1. Goldbach’s conjecture states that every even number greater than 2 can be written as the sum of two primes. For example, $4 = 2 + 2$. Write the next five even numbers as the sum of two primes.

31. The pattern 1, 1, 2, 3, 5, 8, 13, 21, ... is known as the **Fibonacci sequence**. Find the next three terms in the sequence and write a conjecture for the pattern.

32. Look at a monthly calendar and pick any three squares in a row—across, down, or diagonal. Make a conjecture about the number in the middle.

33. Make a conjecture about the value of $2n - 1$ when $n$ is an integer.

34. **Critical Thinking** The turnaround date for migrating gray whales occurs when the number of northbound whales exceeds the number of southbound whales. Make a conjecture about the turnaround date, based on the table below. What factors might affect the validity of your conjecture in the future?

<table>
<thead>
<tr>
<th>Migration Direction of Gray Whales</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Southbound</td>
</tr>
<tr>
<td>Northbound</td>
</tr>
</tbody>
</table>

35. **Write About It** Explain why a true conjecture about even numbers does not necessarily hold for all numbers. Give an example to support your answer.

36. This problem will prepare you for the Multi-Step TAKS Prep on page 102.
   a. For how many hours did the Mock Turtle do lessons on the third day?
   b. On what day did the Mock Turtle do 1 hour of lessons?

“And how many hours a day did you do lessons?” said Alice, in a hurry to change the subject.

“Ten hours the first day,” said the Mock Turtle: “nine the next, and so on.”
37. Which of the following conjectures is false?
   A If \( x \) is odd, then \( x + 1 \) is even.
   B The sum of two odd numbers is even.
   C The difference of two even numbers is positive.
   D If \( x \) is positive, then \(-x\) is negative.

38. A student conjectures that if \( x \) is a prime number, then \( x + 1 \) is not prime. Which of the following is a counterexample?
   F \( x = 11 \)  G \( x = 6 \)  H \( x = 3 \)  I \( x = 2 \)

39. The class of 2004 holds a reunion each year. In 2005, 87.5% of the 120 graduates attended. In 2006, 90 students went, and in 2007, 75 students went. About how many students do you predict will go to the reunion in 2010?
   A 12  B 15  C 24  D 30

**CHALLENGE AND EXTEND**

40. **Multi-Step** Make a table of values for the rule \( x^2 + x + 11 \) when \( x \) is an integer from 1 to 8. Make a conjecture about the type of number generated by the rule. Continue your table. What value of \( x \) generates a counterexample?

41. **Political Science** Presidential elections are held every four years. U.S. senators are elected to 6-year terms, but only \( \frac{1}{3} \) of the Senate is up for election every two years. If \( \frac{1}{3} \) of the Senate is elected during a presidential election year, how many years must pass before these same senate seats are up for election during another presidential election year?

42. **Physical Fitness** Rob is training for the President’s Challenge physical fitness program. During his first week of training, Rob does 15 sit-ups each day. He will add 20 sit-ups to his daily routine each week. His goal is to reach 150 sit-ups per day.
   a. Make a table of the number of sit-ups Rob does each week from week 1 through week 10.
   b. During which week will Rob reach his goal?
   c. Write a conjecture for the number of sit-ups Rob does during week \( n \).

43. **Construction** Draw \( \overline{AB} \). Then construct point \( C \) so that it is not on \( \overline{AB} \) and is the same distance from \( A \) and \( B \). Construct \( \overline{AC} \) and \( \overline{BC} \). Compare \( m\angle CAB \) and \( m\angle CBA \) and compare \( AC \) and \( CB \). Make a conjecture.

**SPIRAL REVIEW**

Determine if the given point is a solution to \( y = 3x - 5 \). *(Previous course)*

44. (1, 8)  45. (-2, -11)  46. (3, 4)  47. (-3.5, 0.5)

Find the perimeter or circumference of each of the following. Leave answers in terms of \( x \). *(Lesson 1-5)*

48. a square whose area is \( x^2 \)  49. a rectangle with dimensions \( x \) and \( 4x - 3 \)
50. a triangle with side lengths of \( x + 2 \)  51. a circle whose area is \( 9\pi x^2 \)

A triangle has vertices \((-1, -1), (0, 1), \) and \((4, 0)\). Find the coordinates for the vertices of the image of the triangle after each transformation. *(Lesson 1-7)*

52. \((x, y) \rightarrow (x, y + 2)\)  53. \((x, y) \rightarrow (x + 4, y - 1)\)
Chapter 2 Geometric Reasoning

Venn Diagrams

Number Theory

Recall that in a Venn diagram, ovals are used to represent each set. The ovals can overlap if the sets share common elements.

The real number system contains an infinite number of subsets. The following chart shows some of them. Other examples of subsets are even numbers, multiples of 3, and numbers less than 6.

<table>
<thead>
<tr>
<th>Set</th>
<th>Description</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural numbers</td>
<td>The counting numbers</td>
<td>1, 2, 3, 4, 5, ...</td>
</tr>
<tr>
<td>Whole numbers</td>
<td>The set of natural numbers and 0</td>
<td>0, 1, 2, 3, 4, ...</td>
</tr>
<tr>
<td>Integers</td>
<td>The set of whole numbers and their opposites</td>
<td>..., −2, −1, 0, 1, 2, ...</td>
</tr>
<tr>
<td>Rational numbers</td>
<td>The set of numbers that can be written as a ratio of integers</td>
<td>−(\frac{3}{4}), 5, −2, 0.5, 0</td>
</tr>
<tr>
<td>Irrational numbers</td>
<td>The set of numbers that cannot be written as a ratio of integers</td>
<td>(\pi), (\sqrt{10}), 8 + (\sqrt{2})</td>
</tr>
</tbody>
</table>

**Example**

Draw a Venn diagram to show the relationship between the set of even numbers and the set of natural numbers.

The set of even numbers includes all numbers that are divisible by 2. This includes natural numbers such as 2, 4, and 6. But even numbers such as −4 and −10 are not natural numbers.

So the set of even numbers includes some, but not all, elements in the set of natural numbers. Similarly, the set of natural numbers includes some, but not all, even numbers.

**Try This**

Draw a Venn diagram to show the relationship between the given sets.

1. natural numbers, whole numbers
2. odd numbers, whole numbers
3. irrational numbers, integers
Why learn this?
To identify a species of butterfly, you must know what characteristics one butterfly species has that another does not.

It is thought that the viceroy butterfly mimics the bad-tasting monarch butterfly to avoid being eaten by birds. By comparing the appearance of the two butterfly species, you can make the following conjecture:

*If a butterfly has a curved black line on its hind wing, then it is a viceroy.*

**Conditional Statements**

By phrasing a conjecture as an if-then statement, you can quickly identify its hypothesis and conclusion.

<table>
<thead>
<tr>
<th>DEFINITION</th>
<th>SYMBOLS</th>
<th>VENN DIAGRAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>A <strong>conditional statement</strong> is a statement that can be written in the form “if ( p ), then ( q ).”</td>
<td>( p \rightarrow q )</td>
<td><img src="image" alt="Venn Diagram" /></td>
</tr>
<tr>
<td>The <strong>hypothesis</strong> is the part ( p ) of a conditional statement following the word if.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The <strong>conclusion</strong> is the part ( q ) of a conditional statement following the word then.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Example 1**

**Identifying the Parts of a Conditional Statement**

Identify the hypothesis and conclusion of each conditional.

**A** If a butterfly has a curved black line on its hind wing, then it is a viceroy.
Hypothesis: A butterfly has a curved black line on its hind wing.
Conclusion: The butterfly is a Viceroy.

**B** A number is an integer if it is a natural number.
Hypothesis: A number is a natural number.
Conclusion: The number is an integer.

1. Identify the hypothesis and conclusion of the statement “A number is divisible by 3 if it is divisible by 6.”

Many sentences without the words *if* and *then* can be written as conditionals. To do so, identify the sentence's hypothesis and conclusion by figuring out which part of the statement depends on the other.
EXAMPLE 2

Writing a Conditional Statement

Write a conditional statement from each of the following.

A. The midpoint \( M \) of a segment bisects the segment.

The midpoint \( M \) of a segment bisects the segment.

Conditional: If \( M \) is the midpoint of a segment, then \( M \) bisects the segment.

B. If an animal is a tarantula, then it is a spider.

The inner oval represents the hypothesis, and the outer oval represents the conclusion.

Conditional: If an animal is a tarantula, then it is a spider.

2. Write a conditional statement from the sentence “Two angles that are complementary are acute.”

A conditional statement has a truth value of either true (T) or false (F). It is false only when the hypothesis is true and the conclusion is false. Consider the conditional “If I get paid, I will take you to the movie.” If I don’t get paid, I haven’t broken my promise. So the statement is still true.

To show that a conditional statement is false, you need to find only one counterexample where the hypothesis is true and the conclusion is false.

EXAMPLE 3

Analyzing the Truth Value of a Conditional Statement

Determine if each conditional is true. If false, give a counterexample.

A. If you live in El Paso, then you live in Texas.

When the hypothesis is true, the conclusion is also true because El Paso is in Texas. So the conditional is true.

B. If an angle is obtuse, then it has a measure of 100°.

You can draw an obtuse angle whose measure is not 100°. In this case, the hypothesis is true, but the conclusion is false. Since you can find a counterexample, the conditional is false.

C. If an odd number is divisible by 2, then 8 is a perfect square.

An odd number is never divisible by 2, so the hypothesis is false. The number 8 is not a perfect square, so the conclusion is false. However, the conditional is true because the hypothesis is false.

3. Determine if the conditional “If a number is odd, then it is divisible by 3” is true. If false, give a counterexample.

The negation of statement \( p \) is “not \( p \),” written as \( \sim p \). The negation of the statement “\( M \) is the midpoint of \( \overline{AB} \)” is “\( M \) is not the midpoint of \( \overline{AB} \).” The negation of a true statement is false, and the negation of a false statement is true. Negations are used to write related conditional statements.
### Related Conditionals

<table>
<thead>
<tr>
<th>DEFINITION</th>
<th>SYMBOLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A conditional is a statement that can be written in the form “If p, then q.”</td>
<td>$p \rightarrow q$</td>
</tr>
<tr>
<td>The <strong>converse</strong> is the statement formed by exchanging the hypothesis and conclusion.</td>
<td>$q \rightarrow p$</td>
</tr>
<tr>
<td>The <strong>inverse</strong> is the statement formed by negating the hypothesis and the conclusion.</td>
<td>$\sim p \rightarrow \sim q$</td>
</tr>
<tr>
<td>The <strong>contrapositive</strong> is the statement formed by both exchanging and negating the hypothesis and conclusion.</td>
<td>$\sim q \rightarrow \sim p$</td>
</tr>
</tbody>
</table>

### Example 4: Biology Application

Write the converse, inverse, and contrapositive of the conditional statement. Use the photos to find the truth value of each.

*If an insect is a butterfly, then it has four wings.*

**Conditional**: If an insect is a butterfly, then it has four wings.

**Converse**: If an insect has four wings, then it is a butterfly.

A moth also is an insect with four wings.

So the converse is false.

**Inverse**: If an insect is not a butterfly, then it does not have four wings.

A moth is not a butterfly, but it has four wings. So the inverse is false.

**Contrapositive**: If an insect does not have four wings, then it is not a butterfly.

Butterflies must have four wings. So the contrapositive is true.

4. Write the converse, inverse, and contrapositive of the conditional statement “If an animal is a cat, then it has four paws.” Find the truth value of each.

In the example above, the conditional statement and its contrapositive are both true, and the converse and inverse are both false. Related conditional statements that have the same truth value are called **logically equivalent statements**. A conditional and its contrapositive are logically equivalent, and so are the converse and inverse.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Example</th>
<th>Truth Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional</td>
<td>If $\angle A = 95^\circ$, then $\angle A$ is obtuse.</td>
<td>T</td>
</tr>
<tr>
<td>Converse</td>
<td>If $\angle A$ is obtuse, then $m\angle A = 95^\circ$.</td>
<td>F</td>
</tr>
<tr>
<td>Inverse</td>
<td>If $m\angle A \neq 95^\circ$, then $\angle A$ is not obtuse.</td>
<td>F</td>
</tr>
<tr>
<td>Contrapositive</td>
<td>If $\angle A$ is not obtuse, then $m\angle A \neq 95^\circ$.</td>
<td>T</td>
</tr>
</tbody>
</table>

However, the converse of a true conditional is not necessarily false. All four related conditionals can be true, or all four can be false, depending on the statement.
GUIDED PRACTICE

**Vocabulary**  Apply the vocabulary from this lesson to answer each question.

1. The ____ of a conditional statement is formed by exchanging the hypothesis and conclusion. (converse, inverse, or contrapositive)

2. A conditional and its contrapositive are ____ because they have the same truth value. (logically equivalent or converses)

**SEE EXAMPLE 1**

Identify the hypothesis and conclusion of each conditional.

3. If a person is at least 16 years old, then the person can drive a car.
4. A figure is a parallelogram if it is a rectangle.
5. The statement $a - b < a$ implies that $b$ is a positive number.

**SEE EXAMPLE 2**

Write a conditional statement from each of the following.

6. Eighteen-year-olds are eligible to vote.
7. $\left(\frac{a}{b}\right)^2 < \frac{a}{b}$ when $0 < a < b$.

**SEE EXAMPLE 3**

Determine if each conditional is true. If false, give a counterexample.

9. If three points form the vertices of a triangle, then they lie in the same plane.
10. If $x > y$, then $|x| > |y|$. 
11. If the season is spring, then the month is March.

**SEE EXAMPLE 4**

12. **Travel** Write the converse, inverse, and contrapositive of the following conditional statement. Find the truth value of each.

   If Brielle drives at exactly 30 mi/h, then she travels 10 mi in 20 min.
Identify the hypothesis and conclusion of each conditional.

13. If an animal is a tabby, then it is a cat.
14. Four angles are formed if two lines intersect.
15. If 8 ounces of cereal cost $2.99, then 16 ounces of cereal cost $5.98.

Write a conditional statement from each sentence.

16. You should monitor the heart rate of a patient who is ill.
17. After three strikes, the batter is out.
18. Congruent segments have equal measures.

Determine if each conditional is true. If false, give a counterexample.

19. If you subtract \(-2\) from \(-6\), then the result is \(-4\).
20. If two planes intersect, then they intersect in exactly one point.
21. If a cat is a bird, then today is Friday.

Write the converse, inverse, and contrapositive of each conditional statement. Find the truth value of each.

22. Probability If the probability of an event is 0.1, then the event is unlikely to occur.
23. Meteorology If freezing rain is falling, then the air temperature is 32°F or less. (Hint: The freezing point of water is 32°F)

Find the truth value of each statement.

24. \(E\) lies in plane \(\mathcal{R}\).
25. \(\overrightarrow{CD}\) lies in plane \(\mathcal{F}\).
26. \(C, E,\) and \(D\) are coplanar.
27. Plane \(\mathcal{F}\) contains \(\overrightarrow{ED}\).
28. \(B\) and \(E\) are collinear.
29. \(\overrightarrow{BC}\) contains \(\mathcal{F}\) and \(\mathcal{R}\).

Draw a Venn diagram.

30. All integers are rational numbers.
31. All natural numbers are real.
32. All rectangles are quadrilaterals.
33. Plane is an undefined term.

Write a conditional statement from each Venn diagram.

34. Mammals
    Dolphins
35. Americans
    Texans
36. \(x < -1\)
    \(x < -4\)

37. This problem will prepare you for the Multi-Step TAKS Prep on page 102.
   a. Identify the hypothesis and conclusion in the Duchess’s statement.
   b. Rewrite the Duchess’s claim as a conditional statement.
Find a counterexample to show that the converse of each conditional is false.

38. If \( x = -5 \), then \( x^2 = 25 \).

39. If two angles are vertical angles, then they are congruent.

40. If two angles are adjacent, then they share a vertex.

41. If you use sunscreen, then you will not get sunburned.

Geology  Mohs’ scale is used to identify minerals.

A mineral with a higher number is harder than a mineral with a lower number.

Use the table and the statements below for Exercises 42–47. Write each conditional and find its truth value.

\[ \begin{align*}
  p &: \text{calcite} \\
  q &: \text{not apatite} \\
  r &: \text{a hardness of 3} \\
  s &: \text{a hardness less than 5}
\end{align*} \]

42. \( p \to r \)  
43. \( s \to q \)  
44. \( q \to s \)  
45. \( q \to p \)  
46. \( r \to q \)  
47. \( p \to s \)

48. Critical Thinking  Consider the conditional

“If two angles are congruent, then they have the same measure.” Write the converse, inverse, and contrapositive and find the truth value of each. Use the related conditionals to draw a Venn diagram that represents the relationship between congruent angles and their measures.

49. Write About It  When is a conditional statement false? Explain why a true conditional statement can have a hypothesis that is false.

50. What is the inverse of “If it is Saturday, then it is the weekend”?  
   \( A \) If it is the weekend, then it is Saturday.  
   \( B \) If it is not Saturday, then it is the weekend.  
   \( C \) If it is not Saturday, then it is not the weekend.  
   \( D \) If it is the weekend, then it is not Saturday.

51. Let \( a \) represent “Two lines are parallel to the same line,” and let \( b \) represent “The two lines are parallel.” Which symbolic statement represents the conditional “If two lines are NOT parallel, then they are parallel to the same line”?
   \( F \) \( a \to b \)  
   \( G \) \( b \to a \)  
   \( H \) \( \sim b \to a \)  
   \( I \) \( b \to \sim a \)

52. Which statement is a counterexample for the conditional statement

“If \( f(x) = \sqrt{25 - x^2} \), then \( f(x) \) is positive”?
   \( A \) \( x = 0 \)  
   \( B \) \( x = 3 \)  
   \( C \) \( x = 4 \)  
   \( D \) \( x = 5 \)

53. Which statement has the same truth value as its converse?
   \( F \) If a triangle has a right angle, its side lengths are 3 centimeters, 4 centimeters, and 5 centimeters.  
   \( G \) If an angle measures 104°, then the angle is obtuse.  
   \( H \) If a number is an integer, then it is a natural number.  
   \( I \) If an angle measures 90°, then it is an acute angle.
**CHALLENGE AND EXTEND**

For each Venn diagram, write two statements beginning with *Some*, *All*, or *No*.

54. Points

55. Students

56. Given: If a figure is a square, then it is a rectangle. Figure $A$ is not a rectangle.

   Conclusion: Figure $A$ is not a square.

   a. Draw a Venn diagram to represent the given conditional statement.

   Use the Venn diagram to explain why the conclusion is valid.

   b. Write the contrapositive of the given conditional. How can you use the contrapositive to justify the conclusion?

57. **Multi-Step** How many true conditionals can you write using the statements below?

   $p$: $n$ is an integer.

   $q$: $n$ is a whole number.

   $r$: $n$ is a natural number.

**SPIRAL REVIEW**

Write a rule to describe each relationship. *(Previous course)*

58. $\begin{array}{c|cl}
     x & -8 & 4 & 7 & 9 \\
     \hline
     y & -5 & 7 & 10 & 12
\end{array}$

59. $\begin{array}{c|cl}
     x & -3 & -1 & 0 & 4 \\
     \hline
     y & -5 & -1 & 1 & 9
\end{array}$

60. $\begin{array}{c|cl}
     x & -2 & 0 & 4 & 6 \\
     \hline
     y & -9 & -4 & 6 & 11
\end{array}$

Determine whether each statement is true or false. If false, explain why. *(Lesson 1-4)*

61. If two angles are complementary and congruent, then the measure of each is $45^\circ$.

62. A pair of acute angles can be supplementary.

63. A linear pair of angles is also a pair of supplementary angles.

Find the next item in each pattern. *(Lesson 2-1)*

64. 1, 13, 131, 1313, ...

65. $\frac{2}{3}, \frac{2}{9}, \frac{2}{27}, ...$

66. $x, 2x^2, 3x^3, 4x^4, ...$

**Career Path**

**Q:** What high school math classes did you take?

**A:** I took three years of math: Pre-Algebra, Algebra, and Geometry.

**Q:** What training do you need to be a desktop publisher?

**A:** Most of my training was done on the job. The computer science and typing classes I took in high school have been helpful.

**Q:** How do you use math?

**A:** Part of my job is to make sure all the text, charts, and photographs are formatted to fit the layout of each page. I have to manipulate things by comparing ratios, calculating areas, and using estimation.

**Q:** What future plans do you have?

**A:** My goal is to start my own business as a freelance graphic artist.

**Stephanie Poulin**

Desktop Publisher

*Daily Reporter*
Why learn this?

You can use inductive and deductive reasoning to decide whether a common myth is accurate.

You learned in Lesson 2-1 that one counterexample is enough to disprove a conjecture. But to prove that a conjecture is true, you must use deductive reasoning. Deductive reasoning is the process of using logic to draw conclusions from given facts, definitions, and properties.

**EXAMPLE 1**

**Media Application**

Urban legends and modern myths spread quickly through the media. Many Web sites and television shows are dedicated to confirming or disproving such myths. Is each conclusion a result of inductive or deductive reasoning?

A

There is a myth that toilets and sinks drain in opposite directions in the Southern and Northern Hemispheres. However, if you were to observe sinks draining in the two hemispheres, you would see that this myth is false. Since the conclusion is based on a pattern of observation, it is a result of inductive reasoning.

B

There is a myth that you should not touch a baby bird that has fallen from its nest because the mother bird will disown the baby if she detects human scent. However, biologists have shown that birds cannot detect human scent. Therefore, the myth cannot be true. The conclusion is based on logical reasoning from scientific research. It is a result of deductive reasoning.

1. There is a myth that an eelskin wallet will demagnetize credit cards because the skin of the electric eels used to make the wallet holds an electric charge. However, eelskin products are not made from electric eels. Therefore, the myth cannot be true. Is this conclusion a result of inductive or deductive reasoning?

In deductive reasoning, if the given facts are true and you apply the correct logic, then the conclusion must be true. The Law of Detachment is one valid form of deductive reasoning.
If \( p \rightarrow q \) is a true statement and \( p \) is true, then \( q \) is true.

**Law of Detachment**

**Example 2**

**Verifying Conjectures by Using the Law of Detachment**

Determine if each conjecture is valid by the Law of Detachment.

A

Given: If two segments are congruent, then they have the same length. \( \overline{AB} \cong \overline{XY} \).

Conjecture: \( AB = XY \)

Identify the hypothesis and conclusion in the given conditional.

If two segments are congruent, then they have the same length.

The given statement \( \overline{AB} \cong \overline{XY} \) matches the hypothesis of a true conditional. By the Law of Detachment \( AB = XY \). The conjecture is valid.

B

Given: If you are tardy 3 times, you must go to detention. Shea is in detention.

Conjecture: Shea was tardy at least 3 times.

Identify the hypothesis and conclusion in the given conditional.

If you are tardy 3 times, you must go to detention.

The given statement “Shea is in detention” matches the conclusion of a true conditional. But this does not mean the hypothesis is true. Shea could be in detention for another reason. The conjecture is not valid.

2. Determine if the conjecture is valid by the Law of Detachment.

Given: If a student passes his classes, the student is eligible to play sports. Ramon passed his classes.

Conjecture: Ramon is eligible to play sports.

Another valid form of deductive reasoning is the Law of Syllogism. It allows you to draw conclusions from two conditional statements when the conclusion of one is the hypothesis of the other.

**Law of Syllogism**

If \( p \rightarrow q \) and \( q \rightarrow r \) are true statements, then \( p \rightarrow r \) is a true statement.

**Example 3**

**Verifying Conjectures by Using the Law of Syllogism**

Determine if each conjecture is valid by the Law of Syllogism.

A

Given: If \( m\angle A < 90^\circ \), then \( \angle A \) is acute. If \( \angle A \) is acute, then it is not a right angle.

Conjecture: If \( m\angle A < 90^\circ \), then it is not a right angle.

Let \( p, q, \) and \( r \) represent the following.

\( p: \) The measure of an angle is less than 90°.

\( q: \) The angle is acute.

\( r: \) The angle is not a right angle.

You are given that \( p \rightarrow q \) and \( q \rightarrow r \). Since \( q \) is the conclusion of the first conditional and the hypothesis of the second conditional, you can conclude that \( p \rightarrow r \). The conjecture is valid by the Law of Syllogism.
Determine if each conjecture is valid by the Law of Syllogism.

B  Given: If a number is divisible by 4, then it is divisible by 2. 
   If a number is even, then it is divisible by 2. 
   Conjecture: If a number is divisible by 4, then it is even. 
   Let \( x, y, \) and \( z \) represent the following. 
   \( x: \text{A number is divisible by 4.} \) 
   \( y: \text{A number is divisible by 2.} \) 
   \( z: \text{A number is even.} \) 
   You are given that \( x \rightarrow y \) and \( z \rightarrow y. \) The Law of Syllogism cannot be 
   used to draw a conclusion since \( y \) is the conclusion of both conditionals. 
   Even though the conjecture \( x \rightarrow z \) is true, the logic used to draw the 
   conclusion is not valid.

3. Determine if the conjecture is valid by the Law of Syllogism. 
   Given: If an animal is a mammal, then it has hair. 
   If an animal is a dog, then it is a mammal. 
   Conjecture: If an animal is a dog, then it has hair.

**Example 4** Applying the Laws of Deductive Reasoning

Draw a conclusion from the given information.

A  Given: If a team wins 10 games, then they play in the finals. If a team 
   plays in the finals, then they travel to Boston. The Ravens won 
   10 games. 
   Conclusion: The Ravens will travel to Boston.

B  Given: If two angles form a linear pair, then they are adjacent. 
   If two angles are adjacent, then they share a side. \( \angle 1 \) and 
   \( \angle 2 \) form a linear pair. 
   Conclusion: \( \angle 1 \) and \( \angle 2 \) share a side.

4. Draw a conclusion from the given information. 
   Given: If a polygon is a triangle, then it has three sides. 
   If a polygon has three sides, then it is not a 
   quadrilateral. Polygon \( P \) is a triangle.

**Think and Discuss**

1. Could “A square has exactly two sides” be the conclusion of a valid 
   argument? If so, what do you know about the truth value of the 
   given information?

2. Explain why writing conditional statements as symbols might help 
   you evaluate the validity of an argument.

3. **Get Organized** Copy and 
   complete the graphic organizer. 
   Write each law in your own 
   words and give an example 
   of each.
**GUIDED PRACTICE**

1. **Vocabulary** Explain how *deductive reasoning* differs from inductive reasoning.

Does each conclusion use inductive or deductive reasoning?

2. At Bell High School, students must take Biology before they take Chemistry. Sam is in Chemistry, so Marcia concludes that he has taken Biology.

3. A detective learns that his main suspect was out of town the day of the crime. He concludes that the suspect is innocent.

Determine if each conjecture is valid by the Law of Detachment.

4. Given: If you want to go on a field trip, you must have a signed permission slip. Zola has a signed permission slip. Conjecture: Zola wants to go on a field trip.

5. Given: If the side lengths of a rectangle are 3 ft and 4 ft, then its area is 12 ft\(^2\). A rectangle has side lengths of 3 ft and 4 ft. Conjecture: The area of the rectangle is 12 ft\(^2\).

Determine if each conjecture is valid by the Law of Syllogism.

6. Given: If you fly from Texas to California, you travel from the central to the Pacific time zone. If you travel from the central to the Pacific time zone, then you gain two hours. Conjecture: If you fly from Texas to California, you gain two hours.

7. Given: If a figure is a *square*, then the figure is a *rectangle*. If a figure is a *square*, then it is a *parallelogram*. Conjecture: If a figure is a *parallelogram*, then it is a *rectangle*.

8. Draw a conclusion from the given information. Given: If you leave your car lights on overnight, then your car battery will drain. If your battery is drained, your car might not start. Alex left his car lights on last night.

**PRACTICE AND PROBLEM SOLVING**

Does each conclusion use inductive or deductive reasoning?

9. The sum of the angle measures of a triangle is 180°. Two angles of a triangle measure 40° and 60°, so Kandy concludes that the third angle measures 80°.

10. All of the students in Henry’s Geometry class are juniors. Alexander takes Geometry, but has another teacher. Henry concludes that Alexander is also a junior.

11. Determine if the conjecture is valid by the Law of Detachment. Given: If one integer is odd and another integer is even, their product is even. The product of two integers is 24. Conjecture: One of the two integers is odd.
12. **Science** Determine if the conjecture is valid by the Law of Syllogism.
   Given: If an element is an alkali metal, then it reacts with water. If an element is in the first column of the periodic table, then it is an alkali metal.
   Conjecture: If an element is in the first column of the periodic table, then it reacts with water.

13. Draw a conclusion from the given information.
   Given: If Dakota watches the news, she is informed about current events.
   If Dakota knows about current events, she gets better grades in Social Studies.
   Dakota watches the news.

14. **Technology** Joseph downloads a file in 18 minutes with a dial-up modem. How long would it take to download the file with a Cheetah-Net cable modem?

Recreation **Use the true statements below for Exercises 15–18. Determine whether each conclusion is valid.**

1. The Top Thrill Dragster is at Cedar Point amusement park in Sandusky, OH.
2. Carter and Mary go to Cedar Point.
3. The Top Thrill Dragster roller coaster reaches speeds of 120 mi/h.
4. When Carter goes to an amusement park, he rides all the roller coasters.

15. Carter went to Sandusky, OH.
16. Mary rode the Top Thrill Dragster.
17. Carter rode a roller coaster that travels 120 mi/h.
18. Mary rode a roller coaster that travels 120 mi/h.

   If $3 - x < 5$, then $x < -2$. If $x < -2$, then $-5x > 10$. Thus, if $3 - x < 5$, then $-5x > 10$.

20. **ERROR ANALYSIS** Below are two conclusions. Which is incorrect? Explain the error.
   If two angles are complementary, their measures add to 90°. If an angle measures 90°, then it is a right angle. $\angle A$ and $\angle B$ are complementary.

21. **Write About It** Write one example of a real-life logical argument that uses the Law of Detachment and one that uses the Law of Syllogism. Explain why the conclusions are valid.

22. **Multi-Step TAKS Prep** This problem will prepare you for the Multi-Step TAKS Prep on page 102.
   When Alice meets the Pigeon in Wonderland, the Pigeon thinks she is a serpent. The Pigeon reasons that serpents eat eggs, and Alice confirms that she has eaten eggs.
   a. Write “Serpents eat eggs” as a conditional statement.
   b. Is the Pigeon’s conclusion that Alice is a serpent valid? Explain your reasoning.
23. The Supershots scored over 75 points in each of ten straight games. The newspaper predicts that they will score more than 75 points tonight. Which form of reasoning is this conclusion based on?
   A. Deductive reasoning, because the conclusion is based on logic
   B. Deductive reasoning, because the conclusion is based on a pattern
   C. Inductive reasoning, because the conclusion is based on logic
   D. Inductive reasoning, because the conclusion is based on a pattern

24. $\overline{HF}$ bisects $\angle EHG$. Which conclusion is NOT valid?
   F. $E$, $F$, and $G$ are coplanar.
   G. $\angle EHF \cong \angle FHG$
   H. $EF \cong FG$
   I. $m\angle EHF = m\angle FHG$

25. **Gridded Response** If Whitney plays a low G on her piano, the frequency of the note is 24.50 hertz. The frequency of a note doubles with each octave. What is the frequency in hertz of a G note that is 3 octaves above low G?

26. **Political Science** To be eligible to hold the office of the president of the United States, a person must be at least 35 years old, be a natural-born U.S. citizen, and have been a U.S. resident for at least 14 years. Given this information, what conclusion, if any, can be drawn from the statements below? Explain your reasoning.

Andre is not eligible to be the president of the United States.
Andre has lived in the United States for 16 years.

27. **Multi-Step** Consider the two conditional statements below.
   If you live in San Diego, then you live in California.
   If you live in California, then you live in the United States.
   a. Draw a conclusion from the given conditional statements.
   b. Write the contrapositive of each conditional statement.
   c. Draw a conclusion from the two contrapositives.
   d. How does the conclusion in part a relate to the conclusion in part c?

28. If Cassie goes to the skate park, Hanna and Amy will go. If Hanna or Amy goes to the skate park, then Marc will go. If Marc goes to the skate park, then Dallas will go. If only two of the five people went to the skate park, who were they?

29. Simplify each expression. *(Previous course)*
   29. $2(x + 5)$
   30. $(4y + 6) - (3y - 5)$
   31. $(3c + 4c) + 2(-7c + 7)$

32. Find the coordinates of the midpoint of the segment connecting each pair of points. *(Lesson 1-6)*
   32. $(1, 2)$ and $(4, 5)$
   33. $(-3, 6)$ and $(0, 1)$
   34. $(-2.5, 9)$ and $(2.5, -3)$

35. Identify the hypothesis and conclusion of each conditional statement. *(Lesson 2-2)*
   35. If the fire alarm rings, then everyone should exit the building.
   36. If two different lines intersect, then they intersect at exactly one point.
   37. The statement $\overline{AB} \cong \overline{CD}$ implies that $AB = CD$. 
Activity 1

Bonnie, Cally, Daphne, and Fiona own a bird, cat, dog, and fish. No girl has a type of pet that begins with the same letter as her name. Bonnie is allergic to animal fur. Daphne feeds Fiona’s bird when Fiona is away. Make a table to determine who owns which animal.

1. Since no girl has a type of pet that starts with the same letter as her name, place an X in each box along the diagonal of the table.

<table>
<thead>
<tr>
<th></th>
<th>Bird</th>
<th>Cat</th>
<th>Dog</th>
<th>Fish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonnie</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cally</td>
<td></td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daphne</td>
<td></td>
<td></td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Fiona</td>
<td></td>
<td></td>
<td></td>
<td>×</td>
</tr>
</tbody>
</table>

2. Bonnie cannot have a cat or dog because of her allergy. So she must own the fish, and no other girl can have the fish.

<table>
<thead>
<tr>
<th></th>
<th>Bird</th>
<th>Cat</th>
<th>Dog</th>
<th>Fish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonnie</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>✓</td>
</tr>
<tr>
<td>Cally</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daphne</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fiona</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Fiona owns the bird, so place a check in Fiona’s row, in the bird column. Place an X in the remaining boxes in the same column and row.

<table>
<thead>
<tr>
<th></th>
<th>Bird</th>
<th>Cat</th>
<th>Dog</th>
<th>Fish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonnie</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>✓</td>
</tr>
<tr>
<td>Cally</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daphne</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fiona</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

4. Therefore, Daphne owns the cat, and Cally owns the dog.

<table>
<thead>
<tr>
<th></th>
<th>Bird</th>
<th>Cat</th>
<th>Dog</th>
<th>Fish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonnie</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>✓</td>
</tr>
<tr>
<td>Cally</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daphne</td>
<td></td>
<td></td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>Fiona</td>
<td>✓</td>
<td>×</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Try This

1. After figuring out that Fiona owns the bird in Step 3, why can you place an X in every other box in that row and column?

2. Ally, Emily, Misha, and Tracy go to a dance with Danny, Frank, Jude, and Kian. Ally and Frank are siblings. Jude and Kian are roommates. Misha does not know Kian. Emily goes with Kian’s roommate. Tracy goes with Ally’s brother. Who went to the dance with whom?
Activity 2

A farmer has a goat, a wolf, and a cabbage. He wants to transport all three from one side of a river to the other. He has a boat, but it has only enough room for the farmer and one thing. The wolf will eat the goat if they are left alone together, and the goat will eat the cabbage if they are left alone. How can the farmer get everything to the other side of the river?

You can use a network to solve this kind of puzzle. A network is a diagram of vertices and edges, also known as a graph. An edge is a curve or a segment that joins two vertices of the graph. A vertex is a point on the graph.

1. Let $F$ represent the farmer, $W$ represent the wolf, $G$ represent the goat, and $C$ represent the cabbage. Use an ordered pair to represent what is on each side of the river. The first ordered pair is $(FWGC, -)$, and the desired result is $(-, FWGC)$.

2. Draw a vertex and label it with the first ordered pair. Then draw an edge and vertex for each possible trip the farmer could make across the river. If at any point a path results in an unworkable combination of things, no more edges can be drawn from that vertex.

3. From each workable vertex, continue to draw edges and vertices that represent the next trip across the river. When you get to a vertex for $(-, FWGC)$, the network is complete.

4. Use the network to write out the solution in words.

Try This

3. What combinations are unworkable? Why?

4. How many solutions are there to the farmer’s transport problem? How many steps does each solution take?

5. What is the advantage of drawing a complete solution network rather than working out one solution with a diagram?

6. Madeline has two measuring cups—a 1-cup measuring cup and a $\frac{3}{4}$-cup measuring cup. Neither cup has any markings on it. How can Madeline get exactly $\frac{1}{2}$ cup of flour in the larger measuring cup? Complete the network below to solve the problem.

\[
\begin{align*}
(0, 0) & \quad (1, 0) \quad \left(0, \frac{3}{4}\right) \\
\end{align*}
\]
2-4

Biconditional Statements and Definitions

Who uses this?
A gardener can plan the color of the hydrangeas she plants by checking the pH of the soil.

The pH of a solution is a measure of the concentration of hydronium ions in the solution. If a solution has a pH less than 7, it is an acid. Also, if a solution is an acid, it has a pH less than 7.

When you combine a conditional statement and its converse, you create a biconditional statement. A biconditional statement is a statement that can be written in the form "p if and only if q." This means "if p, then q" and "if q, then p."

So you can define an acid with the following biconditional statement: A solution is an acid if and only if it has a pH less than 7.

Vocabulary:
- biconditional statement
- definition
- polygon
- triangle
- quadrilateral

EXAMPLE 1

Identifying the Conditionals within a Biconditional Statement

Write the conditional statement and converse within each biconditional.

A. Two angles are congruent if and only if their measures are equal.

Let p and q represent the following.

\[ p: \text{Two angles are congruent.} \]
\[ q: \text{Two angle measures are equal.} \]

The two parts of the biconditional \( p \leftrightarrow q \) are \( p \rightarrow q \) and \( q \rightarrow p \).

Conditional: If two angles are congruent, then their measures are equal.

Converse: If two angle measures are equal, then the angles are congruent.

B. A solution is a base \( \leftrightarrow \) it has a pH greater than 7.

Let \( x \) and \( y \) represent the following.

\[ x: \text{A solution is a base.} \]
\[ y: \text{A solution has a pH greater than 7.} \]

The two parts of the biconditional \( x \leftrightarrow y \) are \( x \rightarrow y \) and \( y \rightarrow x \).

Conditional: If a solution is a base, then it has a pH greater than 7.

Converse: If a solution has a pH greater than 7, then it is a base.

Check it Out!

Write the conditional statement and converse within each biconditional.

1a. An angle is acute iff its measure is greater than 0° and less than 90°.

1b. Cho is a member if and only if he has paid the $5 dues.
**Writing a Biconditional Statement**

For each conditional, write the converse and a biconditional statement.

**A** If \(2x + 5 = 11\), then \(x = 3\).
- Converse: If \(x = 3\), then \(2x + 5 = 11\).
- Biconditional: \(2x + 5 = 11\) if and only if \(x = 3\).

**B** If a point is a midpoint, then it divides the segment into two congruent segments.
- Converse: If a point divides a segment into two congruent segments, then the point is a midpoint.
- Biconditional: A point is a midpoint if and only if it divides the segment into two congruent segments.

**EXAMPLE 2**

For each conditional, write the converse and a biconditional statement.

2a. If the date is July 4th, then it is Independence Day.
2b. If points lie on the same line, then they are collinear.

For a biconditional statement to be true, both the conditional statement and its converse must be true. If either the conditional or the converse is false, then the biconditional statement is false.

**Analyzing the Truth Value of a Biconditional Statement**

Determine if each biconditional is true. If false, give a counterexample.

**A** A square has a side length of 5 if and only if it has an area of 25.
- Conditional: If a square has a side length of 5, then it has an area of 25. *The conditional is true.*
- Converse: If a square has an area of 25, then it has a side length of 5. *The converse is true.*
  Since the conditional and its converse are true, the biconditional is true.

**B** The number \(n\) is a positive integer \(\iff\) \(2n\) is a natural number.
- Conditional: If \(n\) is a positive integer, then \(2n\) is a natural number. *The conditional is true.*
- Converse: If \(2n\) is a natural number, then \(n\) is a positive integer. *The converse is false.*
  If \(2n = 1\), then \(n = \frac{1}{2}\), which is not an integer. Because the converse is false, the biconditional is false.

**EXAMPLE 3**

Determine if each biconditional is true. If false, give a counterexample.

3a. An angle is a right angle \(\iff\) its measure is 90°.
3b. \(y = -5\) \(\iff\) \(y^2 = 25\)

In geometry, biconditional statements are used to write definitions. A definition is a statement that describes a mathematical object and can be written as a true biconditional. Most definitions in the glossary are not written as biconditional statements, but they can be. The “if and only if” is implied.
In the glossary, a **polygon** is defined as a closed plane figure formed by three or more line segments. Each segment intersects exactly two other segments only at their endpoints, and no two segments with a common endpoint are collinear.

<table>
<thead>
<tr>
<th>Polygons</th>
<th>Not Polygons</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Polygons Diagram" /></td>
<td><img src="image2" alt="Not Polygons Diagram" /></td>
</tr>
</tbody>
</table>

A **triangle** is defined as a three-sided polygon, and a **quadrilateral** is a four-sided polygon.

A good, precise definition can be used forward and backward. For example, if a figure is a quadrilateral, then it is a four-sided polygon. If a figure is a four-sided polygon, then it is a quadrilateral. To make sure a definition is precise, it helps to write it as a **biconditional statement**.

**EXAMPLE 4** Writing Definitions as Biconditional Statements

Write each definition as a biconditional.

A. A triangle is a three-sided polygon.  
   A figure is a triangle if and only if it is a three-sided polygon.

B. A segment bisector is a ray, segment, or line that divides a segment into two congruent segments.  
   A ray, segment, or line is a segment bisector if and only if it divides a segment into two congruent segments.

Write each definition as a biconditional.

4a. A quadrilateral is a four-sided polygon.
   4b. The measure of a straight angle is 180°.

**THINK AND DISCUSS**

1. How do you determine if a biconditional statement is true or false?
2. Compare a triangle and a quadrilateral.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Use the definition of a polygon to write a conditional, converse, and biconditional in the appropriate boxes.
GUIDED PRACTICE

1. **Vocabulary**  How is a biconditional statement different from a conditional statement?

Write the conditional statement and converse within each biconditional.

2. Perry can paint the entire living room if and only if he has enough paint.

3. Your medicine will be ready by 5 P.M. if and only if you drop your prescription off by 8 A.M.

For each conditional, write the converse and a biconditional statement.

4. If a student is a sophomore, then the student is in the tenth grade.

5. If two segments have the same length, then they are congruent.

Multi-Step  Determine if each biconditional is true. If false, give a counterexample.

6. \( xy = 0 \leftrightarrow x = 0 \text{ or } y = 0 \).

7. A figure is a quadrilateral if and only if it is a polygon.

Write each definition as a biconditional.

8. Parallel lines are two coplanar lines that never intersect.

9. A hummingbird is a tiny, brightly colored bird with narrow wings, a slender bill, and a long tongue.

PRACTICE AND PROBLEM SOLVING

Write the conditional statement and converse within each biconditional.

10. Three points are coplanar if and only if they lie in the same plane.

11. A parallelogram is a rectangle if and only if it has four right angles.

12. A lunar eclipse occurs if and only if Earth is between the Sun and the Moon.

For each conditional, write the converse and a biconditional statement.

13. If today is Saturday or Sunday, then it is the weekend.

14. If Greg has the fastest time, then he wins the race.

15. If a triangle contains a right angle, then it is a right triangle.

Multi-Step  Determine if each biconditional is true. If false, give a counterexample.

16. Felipe is a swimmer if and only if he is an athlete.

17. The number \( 2n \) is even if and only if \( n \) is an integer.

Write each definition as a biconditional.

18. A circle is the set of all points in a plane that are a fixed distance from a given point.

19. A catcher is a baseball player who is positioned behind home plate and who catches throws from the pitcher.
Algebra Determine if a true biconditional can be written from each conditional statement. If not, give a counterexample.

20. If \( a = b \), then \( |a| = |b| \).
21. If \( 3x - 2 = 13 \), then \( \frac{4}{5}x + 8 = 12 \).
22. If \( y^2 = 64 \), then \( 3y = 24 \).
23. If \( x > 0 \), then \( x^2 > 0 \).

Use the diagrams to write a definition for each figure.

24. Equilateral triangle
25. Square

26. Biology White blood cells are cells that defend the body against invading organisms by engulfing them or by releasing chemicals called antibodies. Write the definition of a white blood cell as a biconditional statement.

Explain why the given statement is not a definition.

27. An automobile is a vehicle that moves along the ground.
28. A calculator is a machine that performs computations with numbers.
29. An angle is a geometric object formed by two rays.

Chemistry Use the table for Exercises 30–32.

Determine if a true biconditional statement can be written from each conditional.

30. If a solution has a pH of 4, then it is tomato juice.
31. If a solution is bleach, then its pH is 13.
32. If a solution has a pH greater than 7, then it is not battery acid.

Complete each statement to form a true biconditional.

33. The circumference of a circle is \( 10\pi \) if and only if its radius is \( ? \).
34. Four points in a plane form a \( ? \) if and only if no three of them are collinear.
35. Critical Thinking Write the definition of a biconditional statement as a biconditional statement. Use the conditional and converse within the statement to explain why your biconditional is true.

36. Write About It Use the definition of an angle bisector to explain what is meant by the statement “A good definition is reversible.”

37. This problem will prepare you for the Multi-Step TAKS Prep on page 102.
   a. Write “I say what I mean” and “I mean what I say” as conditionals.
   b. Explain why the biconditional statement implied by Alice is false.
38. Which is a counterexample for the biconditional “An angle measures 80° if and only if the angle is acute”?
   ① $m\angle S = 60°$  ② $m\angle S = 115°$  ③ $m\angle S = 90°$  ④ $m\angle S = 360°$

39. Which biconditional is equivalent to the spelling phrase “I before E except after C”?
   ① The letter I comes before E if and only if I follows C.
   ② The letter E comes before I if and only if E follows C.
   ③ The letter E comes before I if and only if E comes before C.
   ④ The letter I comes before E if and only if I comes before C.

40. Which conditional statement can be used to write a true biconditional?
   ① If a number is divisible by 4, then it is even.
   ② If a ratio compares two quantities measured in different units, the ratio is a rate.
   ③ If two angles are supplementary, then they are adjacent.
   ④ If an angle is right, then it is not acute.

41. **Short Response** Write the two conditional statements that make up the biconditional “You will get a traffic ticket if and only if you are speeding.” Is the biconditional true or false? Explain your answer.

**CHALLENGE AND EXTEND**

42. **Critical Thinking** Describe what the Venn diagram of a true biconditional statement looks like. How does this support the idea that a definition can be written as a true biconditional?

43. Consider the conditional “If an angle measures 105°, then the angle is obtuse.”
   a. Write the inverse of the conditional statement.
   b. Write the converse of the inverse.
   c. How is the converse of the inverse related to the original conditional?
   d. What is the truth value of the biconditional statement formed by the inverse of the original conditional and the converse of the inverse? Explain.

44. Suppose $A$, $B$, $C$, and $D$ are coplanar, and $A$, $B$, and $C$ are not collinear. What is the truth value of the biconditional formed from the true conditional “If $m\angle ABD + m\angle DBC = m\angle ABC$, then $D$ is in the interior of $\angle ABC$”? Explain.

45. Find a counterexample for “$n$ is divisible by 4 if and only if $n^2$ is even.”

**SPIRAL REVIEW**

Describe how the graph of each function differs from the graph of the parent function $y = x^2$. *(Previous course)*

46. $y = \frac{1}{2}x^2 + 5$  47. $y = -2x^2 - 1$  48. $y = (x - 2)(x + 2)$

A transformation maps $S$ onto $T$ and $X$ onto $Y$. Name each of the following. *(Lesson 1-7)*

49. the image of $S$  50. the image of $X$  51. the preimage of $T$

Determine if each conjecture is true. If not, give a counterexample. *(Lesson 2-1)*

52. If $n \geq 0$, then $\frac{n}{2} > 0$.
53. If $x$ is prime, then $x + 2$ is also prime.
54. The vertices of the image of a figure under the translation $(x, y) \rightarrow (x + 0, y + 0)$ have the same coordinates as the preimage.
Inductive and Deductive Reasoning

Rhyme or Reason

*Alice’s Adventures in Wonderland*

originated as a story told by Charles Lutwidge Dodgson (Lewis Carroll) to three young traveling companions. The story is famous for its wordplay and logical absurdities.

1. When Alice first meets the Cheshire Cat, she asks what sort of people live in Wonderland. The Cat explains that everyone in Wonderland is mad. What conjecture might the Cat make since Alice, too, is in Wonderland?

2. “I don’t much care where—” said Alice.
   “Then it doesn’t matter which way you go,” said the Cat.
   “—so long as I get somewhere,” Alice added as an explanation.
   “Oh, you’re sure to do that,” said the Cat, “if you only walk long enough.”

Write the conditional statement implied by the Cat’s response to Alice.

3. “Well, then,” the Cat went on, “you see a dog growls when it’s angry, and wags its tail when it’s pleased. Now I growl when I’m pleased, and wag my tail when I’m angry. Therefore I’m mad.”

Is the Cat’s conclusion valid by the Law of Detachment or the Law of Syllogism? Explain your reasoning.

4. “You might just as well say,” added the Dormouse, who seemed to be talking in his sleep, “that ‘I breathe when I sleep’ is the same thing as ‘I sleep when I breathe’!”

Write a biconditional statement from the Dormouse’s example. Explain why the biconditional statement is false.
Quiz for Lessons 2-1 Through 2-4

2-1 Using Inductive Reasoning to Make Conjectures

Find the next item in each pattern.

1. 1, 10, 18, 25, …
2. July, May, March, …
3. $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, …
4. $\ldots$

5. A biologist recorded the following data about the weight of male lions in a wildlife park in Africa. Use the table to make a conjecture about the average weight of a male lion.

<table>
<thead>
<tr>
<th>ID Number</th>
<th>Weight (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1902SM</td>
<td>387.2</td>
</tr>
<tr>
<td>A1904SM</td>
<td>420.5</td>
</tr>
<tr>
<td>A1920SM</td>
<td>440.6</td>
</tr>
<tr>
<td>A1956SM</td>
<td>398.7</td>
</tr>
<tr>
<td>A1974SM</td>
<td>415.0</td>
</tr>
</tbody>
</table>

6. Complete the conjecture “The sum of two negative numbers is ______.”

7. Show that the conjecture “If an even number is divided by 2, then the result is an even number” is false by finding a counterexample.

2-2 Conditional Statements

8. Identify the hypothesis and conclusion of the conditional statement “An angle is obtuse if its measure is 107°.”

Write a conditional statement from each of the following.

9. A whole number is an integer.

10. Rectangles

11. The diagonals of a square are congruent.

Determine if each conditional is true. If false, give a counterexample.

12. If an angle is acute, then it has a measure of 30°.

13. If $9x - 11 = 2x + 3$, then $x = 2$.

14. Write the converse, inverse, and contrapositive of the statement “If a number is even, then it is divisible by 4.” Find the truth value of each.

2-3 Using Deductive Reasoning to Verify Conjectures

15. Determine if the following conjecture is valid by the Law of Detachment.

Given: If Sue finishes her science project, she can go to the movie. Sue goes to the movie.

Conjecture: Sue finished her science project.

16. Use the Law of Syllogism to draw a conclusion from the given information.

Given: If one angle of a triangle is 90°, then the triangle is a right triangle. If a triangle is a right triangle, then its acute angle measures are complementary.

2-4 Biconditional Statements and Definitions

17. For the conditional “If two angles are supplementary, the sum of their measures is 180°,” write the converse and a biconditional statement.

18. Determine if the biconditional “$\sqrt{x} = 4$ if and only if $x = 16$” is true. If false, give a counterexample.
A proof is an argument that uses logic, definitions, properties, and previously proven statements to show that a conclusion is true.

If you've ever solved an equation in Algebra, then you've already done a proof! An algebraic proof uses algebraic properties such as the properties of equality and the Distributive Property.

Properties of Equality

<table>
<thead>
<tr>
<th>Property</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition Property of Equality</td>
<td>If ( a = b ), then ( a + c = b + c ).</td>
</tr>
<tr>
<td>Subtraction Property of Equality</td>
<td>If ( a = b ), then ( a - c = b - c ).</td>
</tr>
<tr>
<td>Multiplication Property of Equality</td>
<td>If ( a = b ), then ( ac = bc ).</td>
</tr>
<tr>
<td>Division Property of Equality</td>
<td>If ( a = b ) and ( c \neq 0 ), then ( \frac{a}{c} = \frac{b}{c} ).</td>
</tr>
<tr>
<td>Reflexive Property of Equality</td>
<td>( a = a ).</td>
</tr>
<tr>
<td>Symmetric Property of Equality</td>
<td>If ( a = b ), then ( b = a ).</td>
</tr>
<tr>
<td>Transitive Property of Equality</td>
<td>If ( a = b ) and ( b = c ), then ( a = c ).</td>
</tr>
<tr>
<td>Substitution Property of Equality</td>
<td>If ( a = b ), then ( b ) can be substituted for ( a ) in any expression.</td>
</tr>
</tbody>
</table>

As you learned in Lesson 2-3, if you start with a true statement and each logical step is valid, then your conclusion is valid.

An important part of writing a proof is giving justifications to show that every step is valid. For each justification, you can use a definition, postulate, property, or a piece of information that is given.

**Example 1**

Solving an Equation in Algebra

Solve the equation \(-5 = 3n + 1\). Write a justification for each step.

\[
\begin{align*}
-5 &= 3n + 1 \\
-5 - 1 &= 3n + 1 - 1 \\
-6 &= 3n \\
-\frac{6}{3} &= \frac{3n}{3} \\
-2 &= n \\
\end{align*}
\]

\( n = -2 \) Symmetric Property of Equality

**Check it Out!**

1. Solve the equation \( \frac{1}{2}t = -7 \). Write a justification for each step.
**Problem-Solving Application**

To simulate the motion of an object in a computer game, the designer uses the formula \(sr = 3.6p\) to find the number of pixels the object must travel during each second of animation. In the formula, \(s\) is the desired speed of the object in kilometers per hour, \(r\) is the scale of pixels per meter, and \(p\) is the number of pixels traveled per second.

The graphics in a game are based on a scale of 6 pixels per meter. The designer wants to simulate a vehicle moving at 75 km/h. How many pixels must the vehicle travel each second? Solve the equation for \(p\) and justify each step.

1. **Understand the Problem**

   The answer will be the number of pixels traveled per second.

   List the important information:
   - \(sr = 3.6p\)
   - \(s = 75\) km/h
   - \(p\): pixels traveled per second
   - \(r = 6\) pixels per meter

2. **Make a Plan**

   Substitute the given information into the formula and solve.

3. **Solve**

   \[
   \begin{align*}
   sr &= 3.6p \quad \text{Given equation} \\
   (75)(6) &= 3.6p \quad \text{Substitution Property of Equality} \\
   450 &= 3.6p \quad \text{Simplify.} \\
   \frac{450}{3.6} &= \frac{3.6p}{3.6} \quad \text{Division Property of Equality} \\
   125 &= p \quad \text{Simplify.} \\
   \end{align*}
   \]

   \(p = 125\) pixels

   Symmetric Property of Equality

4. **Look Back**

   Check your answer by substituting it back into the original formula.

   \[
   \begin{align*}
   sr &= 3.6p \\
   (75)(6) &= 3.6(125) \\
   450 &= 450 \quad \checkmark
   \end{align*}
   \]

2. **What is the temperature in degrees Celsius \(C\) when it is 86°F?**

   Solve the equation \(C = \frac{5}{9}(F - 32)\) for \(C\) and justify each step.

   Like algebra, geometry also uses numbers, variables, and operations. For example, segment lengths and angle measures are numbers. So you can use these same properties of equality to write algebraic proofs in geometry.
EXAMPLE 3

Solving an Equation in Geometry

Write a justification for each step.

\[ KM = KL + LM \]
\[ 5x - 4 = (x + 3) + (2x - 1) \]
\[ 5x - 4 = 3x + 2 \]
\[ 2x - 4 = 2 \]
\[ 2x = 6 \]
\[ x = 3 \]

Segment Addition Postulate
Substitution Property of Equality
Simplify.
Subtraction Property of Equality
Addition Property of Equality
Division Property of Equality

3. Write a justification for each step.

\[ m\angle ABC = m\angle ABD + m\angle DBC \]
\[ 8x^\circ = (3x + 5)^\circ + (6x - 16)^\circ \]
\[ 8x = 9x - 11 \]
\[ -x = -11 \]
\[ x = 11 \]

You learned in Chapter 1 that segments with equal lengths are congruent and that angles with equal measures are congruent. So the Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence.

Properties of Congruence

<table>
<thead>
<tr>
<th>SYMBOLS</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive Property of Congruence</td>
<td>figure ( A \cong figure A ) (Reflex. Prop. of ( \cong ))</td>
</tr>
<tr>
<td>Symmetric Property of Congruence</td>
<td>If figure ( A \cong figure B ), then figure ( B \cong figure A ). (Sym. Prop. of ( \cong ))</td>
</tr>
<tr>
<td>Transitive Property of Congruence</td>
<td>If figure ( A \cong figure B ) and figure ( B \cong figure C ), then figure ( A \cong figure C ). (Trans. Prop. of ( \cong ))</td>
</tr>
</tbody>
</table>

EXAMPLE 4

Identifying Properties of Equality and Congruence

Identify the property that justifies each statement.

A. \( m\angle 1 = m\angle 1 \)  
   Reflex. Prop. of \( = \)

B. \( \overline{XY} \cong \overline{VW} \), so \( \overline{VW} \cong \overline{XY} \).  
   Sym. Prop. of \( \cong \)

C. \( \angle ABC \cong \angle ABC \)  
   Reflex. Prop. of \( \cong \)

D. \( \angle 1 \cong \angle 2 \), and \( \angle 2 \cong \angle 3 \). So \( \angle 1 \cong \angle 3 \).  
   Trans. Prop. of \( \cong \)

Identify the property that justifies each statement.

4a. \( DE = GH \), so \( GH = DE \).  
4b. \( 94^\circ = 94^\circ \)
4c. \( 0 = a \), and \( a = x \). So \( 0 = x \).  
4d. \( \angle A \cong \angle Y \), so \( \angle Y \cong \angle A \).
1. **Vocabulary** Write the definition of *proof* in your own words.

2. **Multi-Step** Solve each equation. Write a justification for each step.
   
   2. \( y + 1 = 5 \)
   
   3. \( t - 3.2 = -8.3 \)
   
   4. \( 2p - 30 = -4p + 6 \)
   
   5. \( \frac{x + 3}{2} = 8 \)
   
   6. \( \frac{1}{2}n = \frac{3}{4} \)
   
   7. \( 0 = 2(r - 3) + 4 \)

3. **Nutrition** Amy’s favorite breakfast cereal has 102 Calories per serving. The equation \( C = 9f + 90 \) relates the grams of fat \( f \) in one serving to the Calories \( C \) in one serving. How many grams of fat are in one serving of the cereal? Solve the equation for \( f \) and justify each step.

4. **Movie Rentals** The equation \( C = 5.75 + 0.89m \) relates the number of movie rentals \( m \) to the monthly cost \( C \) of a movie club membership. How many movies did Elias rent this month if his membership cost $11.98? Solve the equation for \( m \) and justify each step.

5. **Identify the property that justifies each statement.**

   12. \( \overline{AB} \cong \overline{AB} \)
   
   13. \( m\angle 1 = m\angle 2 \), and \( m\angle 2 = m\angle 4 \). So \( m\angle 1 = m\angle 4 \).
   
   14. \( x = y \), so \( y = x \).
   
   15. \( \overline{ST} \cong \overline{YZ} \), and \( \overline{YZ} \cong \overline{PR} \). So \( \overline{ST} \cong \overline{PR} \).
PRACTICE AND PROBLEM SOLVING

Multi-Step Solve each equation. Write a justification for each step.

16. \(5x - 3 = 4(x + 2)\)  
17. \(1.6 = 3.2n\)  
18. \(\frac{z}{3} - 2 = -10\)

19. \(- (h + 3) = 72\)  
20. \(9y + 17 = -19\)  
21. \(\frac{1}{2}(p - 16) = 13\)

22. Ecology The equation \(T = 0.03c + 0.05b\) relates the numbers of cans \(c\) and bottles \(b\) collected in a recycling rally to the total dollars \(T\) raised. How many cans were collected if $147 was raised and 150 bottles were collected? Solve the equation for \(c\) and justify each step.

Write a justification for each step.

23. \(m\angle XYZ = m\angle 2 + m\angle 3\)  
24. \(m\angle WYV = m\angle 1 + m\angle 2\)

\[
\begin{align*}
4n - 6 &= 58 + (2n - 12) \\
4n - 6 &= 2n + 46 \\
2n - 6 &= 46 \\
2n &= 52 \\
n &= 26
\end{align*}
\]

Identify the property that justifies each statement.

25. \(KL \cong PR\), so \(PR \cong KL\).  
26. 412 = 412

27. If \(a = b\) and \(b = 0\), then \(a = 0\).  
28. figure \(A \cong \) figure \(A\)

29. Estimation Round the numbers in the equation \(2(3.1x - 0.87) = 94.36\) to the nearest whole number and estimate the solution. Then solve the equation, justifying each step. Compare your estimate to the exact solution.

Use the indicated property to complete each statement.

30. Reflexive Property of Equality: \(3x - 1 = \) ?

31. Transitive Property of Congruence: If \(\angle A \cong \angle X\) and \(\angle X \cong \angle T\), then \(\) ? .

32. Symmetric Property of Congruence: If \(BC \cong NP\), then \(\) ? .

33. Recreation The north campground is midway between the Northpoint Overlook and the waterfall. Use the midpoint formula to find the values of \(x\) and \(y\), and justify each step.

34. Business A computer repair technician charges $35 for each job plus $21 per hour of labor and 110% of the cost of parts. The total charge for a 3-hour job was $169.50. What was the cost of parts for this job? Write and solve an equation and justify each step in the solution.

35. Finance Morgan spent a total of $1,733.65 on her car last year. She spent $92.50 on registration, $79.96 on maintenance, and $983 on insurance. She spent the remaining money on gas. She drove a total of 10,820 miles.

a. How much on average did the gas cost per mile? Write and solve an equation and justify each step in the solution.

b. What if…? Suppose Morgan’s car averages 32 miles per gallon of gas. How much on average did Morgan pay for a gallon of gas?

36. Critical Thinking Use the definition of segment congruence and the properties of equality to show that all three properties of congruence are true for segments.
37. This problem will prepare you for the Multi-Step TAKS Prep on page 126. Recall from Algebra 1 that the Multiplication and Division Properties of Inequality tell you to reverse the inequality sign when multiplying or dividing by a negative number.

a. Solve the inequality $x + 15 \leq 63$ and write a justification for each step.

b. Solve the inequality $-2x > 36$ and write a justification for each step.

38. **Write About It** Compare the conclusion of a deductive proof and a conjecture based on inductive reasoning.

39. Which could NOT be used to justify the statement $\overline{AB} \cong \overline{CD}$?

- A. Definition of congruence
- B. Reflexive Property of Congruence
- C. Symmetric Property of Congruence
- D. Transitive Property of Congruence

40. A club membership costs $35 plus $3 each time $t$ the member uses the pool. Which equation represents the total cost $C$ of the membership?

- F. $35 = C + 3t$
- G. $C + 35 = 3t$
- H. $C = 35 + 3t$
- I. $C = 35t + 3$

41. Which statement is true by the Reflexive Property of Equality?

- A. $x = 35$
- B. $\overline{CD} = \overline{CD}$
- C. $\overline{RT} \cong \overline{TR}$
- D. $\overline{CD} = \overline{CD}$

42. **Gridded Response** In the triangle, $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$. If $m\angle 3 = 2m\angle 1$ and $m\angle 1 = m\angle 2$, find $m\angle 3$ in degrees.

43. In the gate, $PA = QB$, $QB = RA$, and $PA = 18$ in. Find $PR$, and justify each step.

44. **Critical Thinking** Explain why there is no Addition Property of Congruence.

45. **Algebra** Justify each step in the solution of the inequality $7 - 3x > 19$.

46. The members of a high school band have saved $600 for a trip. They deposit the money in a savings account. What additional information is needed to find the amount of interest the account earns during a 3-month period? (Previous course)

47. Use a compass and straightedge to construct each of the following. (Lesson 1-2)

- $\overline{JK}$ congruent to $\overline{MN}$
- A segment bisector of $\overline{JK}$

48. Identify whether each conclusion uses inductive or deductive reasoning. (Lesson 2-3)

- A triangle is obtuse if one of its angles is obtuse. Jacob draws a triangle with two acute angles and one obtuse angle. He concludes that the triangle is obtuse.

- Tonya studied 3 hours for each of her last two geometry tests. She got an A on both tests. She concludes that she will get an A on the next test if she studies for 3 hours.
Who uses this?
To persuade your parents to increase your allowance, your argument must be presented logically and precisely.

When writing a geometric proof, you use deductive reasoning to create a chain of logical steps that move from the hypothesis to the conclusion of the conjecture you are proving. By proving that the conclusion is true, you have proven that the original conjecture is true.

When writing a proof, it is important to justify each logical step with a reason. You can use symbols and abbreviations, but they must be clear enough so that anyone who reads your proof will understand them.

**EXAMPLE 1**

Write a justification for each step, given that \( \angle A \) and \( \angle B \) are complementary and \( \angle A \cong \angle C \).

1. \( \angle A \) and \( \angle B \) are complementary. **Given information**
2. \( m\angle A + m\angle B = 90^\circ \) **Def. of comp. \( \angle \)**
3. \( \angle A \cong \angle C \) **Given information**
4. \( m\angle A = m\angle C \) **Def. of \( \cong \angle \)**
5. \( m\angle C + m\angle B = 90^\circ \) **Subst. Prop. of \( = \)** **Steps 2, 4**
6. \( \angle C \) and \( \angle B \) are complementary. **Def. of comp. \( \angle \)**

**CHECK IT OUT!**

1. Write a justification for each step, given that \( B \) is the midpoint of \( AC \) and \( AB \cong EF \).
   1. \( B \) is the midpoint of \( AC \).
   2. \( AB \cong BC \)
   3. \( AB \cong EF \)
   4. \( BC \cong EF \)

A **theorem** is any statement that you can prove. Once you have proven a theorem, you can use it as a reason in later proofs.

**Theorem**

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-6-1 <strong>Linear Pair Theorem</strong></td>
<td>If two angles form a linear pair, then they are supplementary.</td>
<td>( \angle A ) and ( \angle B ) are supplementary.</td>
</tr>
</tbody>
</table>
### Theorem

#### 2-6-2 Congruent Supplements Theorem

**Theorem**

If two angles are supplementary to the same angle (or to two congruent angles), then the two angles are congruent.

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-6-2</td>
<td>∠1 and ∠2 are supplementary. ∠2 and ∠3 are supplementary.</td>
<td>∠1 ≅ ∠3</td>
</tr>
</tbody>
</table>

A geometric proof begins with *Given* and *Prove* statements, which restate the hypothesis and conclusion of the conjecture. In a **two-column proof**, you list the steps of the proof in the left column. You write the matching reason for each step in the right column.

#### Example 2 Completing a Two-Column Proof

Fill in the blanks to complete a two-column proof of the Linear Pair Theorem.

**Given:** ∠1 and ∠2 form a linear pair.

**Prove:** ∠1 and ∠2 are supplementary.

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ∠1 and ∠2 form a linear pair.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. BA and BC form a line.</td>
<td>2. Def. of lin. pair</td>
</tr>
<tr>
<td>3. m∠ABC = 180°</td>
<td>3. Def. of straight ∠</td>
</tr>
<tr>
<td>5. b. ___?</td>
<td>5. Subst. <em>Steps 3, 4</em></td>
</tr>
<tr>
<td>6. ∠1 and ∠2 are supplementary.</td>
<td>6. c. ___?</td>
</tr>
</tbody>
</table>

Use the existing statements and reasons in the proof to fill in the blanks.

- **a.** m∠1 + m∠2 = m∠ABC  
  *The ∠ Add. Post. is given as the reason.*
- **b.** m∠1 + m∠2 = 180°  
  *Substitute 180° for m∠ABC.*
- **c.** Def. of supp. ∆  
  *The measures of supp. ∆ add to 180° by def.*

#### 2. Fill in the blanks to complete a two-column proof of one case of the Congruent Supplements Theorem.

**Given:** ∠1 and ∠2 are supplementary, and ∠2 and ∠3 are supplementary.

**Prove:** ∠1 ≅ ∠3

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. a. ___?</td>
<td>1. Given</td>
</tr>
</tbody>
</table>
| 2. m∠1 + m∠2 = 180°  
  m∠2 + m∠3 = 180° | 2. Def. of supp. ∆ |
| 4. m∠2 = m∠2 | 4. Reflex. Prop. of = |
| 5. m∠1 = m∠3 | 5. c. ___? |
| 6. d. ___? | 6. Def. of ≅ ∆ |
Before you start writing a proof, you should plan out your logic. Sometimes you will be given a plan for a more challenging proof. This plan will detail the major steps of the proof for you.

### Theorems

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2-6-3 Right Angle Congruence</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theorem</td>
<td>All right angles are congruent.</td>
<td>∠A ≅ ∠B</td>
</tr>
<tr>
<td><strong>2-6-4 Congruent Complements</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theorem</td>
<td>If two angles are complementary to the same angle (or to two congruent angles), then the two angles are congruent.</td>
<td>∠1 ≅ ∠3</td>
</tr>
</tbody>
</table>

### Example 3

**Writing a Two-Column Proof from a Plan**

Use the given plan to write a two-column proof of the Right Angle Congruence Theorem.

**Given:** ∠1 and ∠2 are right angles.

**Prove:** ∠1 ≅ ∠2

**Plan:** Use the definition of a right angle to write the measure of each angle. Then use the Transitive Property and the definition of congruent angles.

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ∠1 and ∠2 are right angles.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. m∠1 = 90°, m∠2 = 90°</td>
<td>2. Def. of rt. ∠</td>
</tr>
<tr>
<td>3. m∠1 = m∠2</td>
<td>3. Trans. Prop. of =</td>
</tr>
<tr>
<td>4. ∠1 ≅ ∠2</td>
<td>4. Def. of ≅ ∠</td>
</tr>
</tbody>
</table>

3. Use the given plan to write a two-column proof of one case of the Congruent Complements Theorem.

**Given:** ∠1 and ∠2 are complementary, and ∠2 and ∠3 are complementary.

**Prove:** ∠1 ≅ ∠3

**Plan:** The measures of complementary angles add to 90° by definition. Use substitution to show that the sums of both pairs are equal. Use the Subtraction Property and the definition of congruent angles to conclude that ∠1 ≅ ∠3.

### The Proof Process

1. Write the conjecture to be proven.
2. Draw a diagram to represent the hypothesis of the conjecture.
3. State the given information and mark it on the diagram.
4. State the conclusion of the conjecture in terms of the diagram.
5. Plan your argument and prove the conjecture.
GuIaded Practice

Vocabulary  Apply the vocabulary from this lesson to answer each question.
1. In a two-column proof, you list the ___?___ in the left column and the ___?___ in the right column. (statements or reasons)
2. A ___?___ is a statement you can prove. (postulate or theorem)
3. Write a justification for each step, given that \( m\angle A = 60^\circ \) and \( m\angle B = 2m\angle A \).
   1. \( m\angle A = 60^\circ \), \( m\angle B = 2m\angle A \)
   2. \( m\angle B = 2(60^\circ) \)
   3. \( m\angle B = 120^\circ \)
   4. \( m\angle A + m\angle B = 60^\circ + 120^\circ \)
   5. \( m\angle A + m\angle B = 180^\circ \)
   6. \( \angle A \) and \( \angle B \) are supplementary.
4. Fill in the blanks to complete the two-column proof.
   Given: \( \angle 2 \equiv \angle 3 \)
   Prove: \( \angle 1 \) and \( \angle 3 \) are supplementary.
   Proof:
<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 2 \equiv \angle 3 )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle 2 = m\angle 3 )</td>
<td>2. a. <em><strong>?</strong></em></td>
</tr>
<tr>
<td>4. ( m\angle 1 + m\angle 2 = 180^\circ )</td>
<td>4. Def. of supp. ( \triangle )</td>
</tr>
<tr>
<td>5. ( m\angle 1 + m\angle 3 = 180^\circ )</td>
<td>5. c. <em><strong>?</strong></em></td>
</tr>
<tr>
<td>6. d. <em><strong>?</strong></em></td>
<td>6. Def. of supp. ( \triangle )</td>
</tr>
</tbody>
</table>

5. Use the given plan to write a two-column proof.
   Given: \( X \) is the midpoint of \( \overline{AY} \), and \( Y \) is the midpoint of \( \overline{XB} \).
   Prove: \( \overline{AX} \equiv \overline{YB} \)
   Plan: By the definition of midpoint, \( \overline{AX} \equiv \overline{XY} \), and \( \overline{XY} \equiv \overline{YB} \).
   Use the Transitive Property to conclude that \( \overline{AX} \equiv \overline{YB} \).
**Independent Practice**

**For Exercises** | **See Example**
--- | ---
6 | 1
7-8 | 2
9-10 | 3

---

**PRACTICE AND PROBLEM SOLVING**

6. Write a justification for each step, given that \( \overline{BX} \) bisects \( \angle ABC \) and \( \angle XBC = 45^\circ \).

1. \( \overline{BX} \) bisects \( \angle ABC \).
2. \( \angle ABX \cong \angle XBC \)
3. \( m\angle ABX = m\angle XBC \)
4. \( m\angle XBC = 45^\circ \)
5. \( m\angle ABX = 45^\circ \)
6. \( m\angle ABX + m\angle XBC = m\angle ABC \)
7. \( 45^\circ + 45^\circ = m\angle ABC \)
8. \( 90^\circ = m\angle ABC \)
9. \( \angle ABC \) is a right angle.

---

Fill in the blanks to complete each two-column proof.

7. **Given:** \( \angle 1 \) and \( \angle 2 \) are supplementary, and \( \angle 3 \) and \( \angle 4 \) are supplementary. 

   **Prove:** \( \angle 1 \cong \angle 4 \)

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 ) and ( \angle 2 ) are supplementary. ( \angle 3 ) and ( \angle 4 ) are supplementary.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. a. ( \angle 2 \cong \angle 3 )</td>
<td>2. Def. of supp. ( \angle )</td>
</tr>
<tr>
<td>3. ( m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4 )</td>
<td>3. b. ( \angle 2 \cong \angle 3 )</td>
</tr>
<tr>
<td>4. ( \angle 2 \cong \angle 3 )</td>
<td>4. Given</td>
</tr>
<tr>
<td>5. ( m\angle 2 = m\angle 3 )</td>
<td>5. Def. of ( \cong )</td>
</tr>
<tr>
<td>6. c. ( \angle 1 \cong \angle 4 )</td>
<td>6. Subtr. Prop. of ( \angle = ) Steps 3, 5</td>
</tr>
<tr>
<td>7. d. ( \angle 1 \cong \angle 4 )</td>
<td>7. d. ( \angle 1 \cong \angle 4 )</td>
</tr>
</tbody>
</table>

---

8. **Given:** \( \angle BAC \) is a right angle. \( \angle 2 \cong \angle 3 \)

   **Prove:** \( \angle 1 \) and \( \angle 3 \) are complementary.

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle BAC ) is a right angle.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle BAC = 90^\circ )</td>
<td>2. a. ( \angle 2 \cong \angle 3 )</td>
</tr>
<tr>
<td>3. ( \angle 2 \cong \angle 3 )</td>
<td>3. Add. Post.</td>
</tr>
<tr>
<td>4. ( m\angle 1 + m\angle 2 = 90^\circ )</td>
<td>4. Subst. Steps 2, 3</td>
</tr>
<tr>
<td>5. ( \angle 2 \cong \angle 3 )</td>
<td>5. Given</td>
</tr>
<tr>
<td>6. ( m\angle 3 )</td>
<td>6. Def. of ( \cong )</td>
</tr>
<tr>
<td>7. ( \angle 1 + m\angle 3 = 90^\circ )</td>
<td>7. d. ( \angle 1 \cong \angle 3 ) Steps 4, 6</td>
</tr>
<tr>
<td>8. e. ( \text{?} )</td>
<td>8. Def. of comp. ( \angle )</td>
</tr>
</tbody>
</table>

---

Use the given plan to write a two-column proof.

9. **Given:** \( \overline{BE} \cong \overline{CE}, \overline{DE} \cong \overline{AE} \)

   **Prove:** \( \overline{AB} \cong \overline{CD} \)

**Plan:** Use the definition of congruent segments to write the given information in terms of lengths. Then use the Segment Addition Postulate to show that \( AB = CD \) and thus \( AB \cong CD \).
Use the given plan to write a two-column proof.

10. Given: \( \angle 1 \) and \( \angle 3 \) are complementary, and \( \angle 2 \) and \( \angle 4 \) are complementary. \( \angle 3 \cong \angle 4 \)

   Prove: \( \angle 1 \cong \angle 2 \)

   Plan: Since \( \angle 1 \) and \( \angle 3 \) are complementary and \( \angle 2 \) and \( \angle 4 \) are complementary, both pairs of angle measures add to 90°. Use substitution to show that the sums of both pairs are equal. Since \( \angle 3 \cong \angle 4 \), their measures are equal. Use the Subtraction Property of Equality and the definition of congruent angles to conclude that \( \angle 1 \cong \angle 2 \).

11. \( m\angle 1 \)

12. \( m\angle 2 \)

13. \( m\angle 3 \)

14. **Engineering** The Bluff Dale Bridge is 140 feet long and spans the Paluxy River in Bluff Dale, Texas. If \( \angle 1 \cong \angle 2 \), which theorem can you use to conclude that \( \angle 3 \cong \angle 4 \)?

15. **Critical Thinking** Explain why there are two cases to consider when proving the Congruent Supplements Theorem and the Congruent Complements Theorem.

16. An angle and its complement are congruent.

17. A pair of right angles forms a linear pair.

18. An angle and its complement form a right angle.

19. A linear pair of angles is complementary.

20. \( (4n + 5)° \) \( (8n - 5)° \)

21. \( (9x - 6)° \) \( (8.5x + 2)° \)

22. \( 4z^2 \) \( (3z + 6)^° \)

23. **Write About It** How are a theorem and a postulate alike? How are they different?

24. **Multi-Step TAKS Prep**

   Sometimes you may be asked to write a proof without a specific statement of the Given and Prove information being provided for you. For each of the following situations, use the triangle to write a Given and Prove statement.

   a. The segment connecting the midpoints of two sides of a triangle is half as long as the third side.

   b. The acute angles of a right triangle are complementary.

   c. In a right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse.
25. Which theorem justifies the conclusion that \( \angle 1 \equiv \angle 4 \)?
   - A. Linear Pair Theorem
   - B. Congruent Supplements Theorem
   - C. Congruent Complements Theorem
   - D. Right Angle Congruence Theorem

26. What can be concluded from the statement \( m\angle 1 + m\angle 2 = 180^\circ \)?
   - F. \( \angle 1 \) and \( \angle 2 \) are congruent.
   - H. \( \angle 1 \) and \( \angle 2 \) are complementary.
   - G. \( \angle 1 \) and \( \angle 2 \) are supplementary.
   - I. \( \angle 1 \) and \( \angle 2 \) form a linear pair.

27. Given: Two angles are complementary. The measure of one angle is 10\(^\circ\) less than the measure of the other angle. Conclusion: The measures of the angles are 85\(^\circ\) and 95\(^\circ\). Which statement is true?
   - A. The conclusion is correct because 85\(^\circ\) is 10\(^\circ\) less than 95\(^\circ\).
   - B. The conclusion is verified by the first statement given.
   - C. The conclusion is invalid because the angles are not congruent.
   - D. The conclusion is contradicted by the first statement given.

**CHALLENGE AND EXTEND**

28. Write a two-column proof.
   **Given:** \( m\angle LAN = 30^\circ \), \( m\angle 1 = 15^\circ \)
   **Prove:** \( \overrightarrow{AM} \) bisects \( \angle LAN \).

   **Multi-Step** Find the value of the variable and the measure of each angle.

29. \( (3a + 1.5)^\circ \)
30. \( (2.5a - 5)^\circ \)

31. What percent of the tires had mileage between 40,000 and 49,999 when replaced?
32. If the company replaces twice as many tires next week, about how many tires would you expect to have lasted between 80,000 and 89,999 miles?

**SPIRAL REVIEW**

The table shows the number of tires replaced by a repair company during one week, classified by the mileage on the tires when they were replaced.

<table>
<thead>
<tr>
<th>Mileage on Replaced Tires</th>
<th>Tires</th>
</tr>
</thead>
<tbody>
<tr>
<td>40,000–49,999</td>
<td>60</td>
</tr>
<tr>
<td>50,000–59,999</td>
<td>82</td>
</tr>
<tr>
<td>60,000–69,999</td>
<td>54</td>
</tr>
<tr>
<td>70,000–79,999</td>
<td>40</td>
</tr>
<tr>
<td>80,000–89,999</td>
<td>14</td>
</tr>
</tbody>
</table>

Use the table for Exercises 31 and 32. *(Previous course)*

33. Through any two collinear points, there is more than one plane containing them.
34. A pair of opposite rays forms a line.

Identify the property that justifies each statement. *(Lesson 2-5)*

35. \( \overrightarrow{JK} \equiv \overrightarrow{KL} \), so \( \overrightarrow{KL} \equiv \overrightarrow{JK} \).
36. If \( m = n \) and \( n = p \), then \( m = p \).
Design Plans for Proofs

Sometimes the most challenging part of writing a proof is planning the logical steps that will take you from the Given statement to the Prove statement. Like working a jigsaw puzzle, you can start with any piece. Write down everything you know from the Given statement. If you don’t see the connection right away, start with the Prove statement and work backward. Then connect the pieces into a logical order.

**Activity**

Prove the Common Angles Theorem.

Given: \( \triangle AXB \cong \triangle CXD \)

Prove: \( \triangle AXC \cong \triangle BXD \)

1. Start by considering the difference in the Given and Prove statements. How does \( \triangle AXB \) compare to \( \triangle AXC \)? How does \( \triangle CXD \) compare to \( \triangle BXD \)?

   *In both cases, \( \triangle BXC \) is combined with the first angle to get the second angle.*

2. The situation involves combining adjacent angle measures, so list any definitions, properties, postulates, and theorems that might be helpful.

   *Definition of congruent angles, Angle Addition Postulate, properties of equality, and Reflexive, Symmetric, and Transitive Properties of Congruence*

3. Start with what you are given and what you are trying to prove and then work toward the middle.

   \[
   \begin{align*}
   \angle AXB & \cong \angle CXD & \text{The first reason will be “Given.”} \\
   m\angle AXB &= m\angle CXD & \text{Def. of } \cong \triangle \\
   ??? &= ??? & \\
   m\angle AXC &= m\angle BXD & ??? \\
   \angle AXC & \cong \angle BXD & \text{The last statement will be the Prove statement.}
   \end{align*}
   \]

4. Based on Step 1, \( \angle BXC \) is the missing piece in the middle of the logical flow. So write down what you know about \( \angle BXC \).

   \[
   \begin{align*}
   \angle BXC & \cong \angle BXC & \text{Reflex. Prop. of } \cong \\
   m\angle BXC &= m\angle BXC & \text{Reflex. Prop. of } =
   \end{align*}
   \]

5. Now you can see that the Angle Addition Postulate needs to be used to complete the proof.

   \[
   \begin{align*}
   m\angle AXB + m\angle BXC &= m\angle AXC & \triangle \text{Add. Post.} \\
   m\angle BXC + m\angle CXD &= m\angle BXD & \triangle \text{Add. Post.}
   \end{align*}
   \]

6. Use the pieces to write a complete two-column proof of the Common Angles Theorem.

**Try This**

1. Describe how a plan for a proof differs from the actual proof.

2. Write a plan and a two-column proof.

   **Given:** \( \overline{BD} \) bisects \( \angle ABC \).
   **Prove:** \( 2m\angle 1 = m\angle ABC \)

3. Write a plan and a two-column proof.

   **Given:** \( \angle LNX \) is a right angle.
   **Prove:** \( \angle 1 \) and \( \angle 2 \) are complementary.
Why learn this?
Flowcharts make it easy to see how the steps of a process are linked together.

A second style of proof is a **flowchart proof**, which uses boxes and arrows to show the structure of the proof. The steps in a flowchart proof move from left to right or from top to bottom, shown by the arrows connecting each box. The justification for each step is written below the box.

**Theorem 2-7-1**

**Common Segments Theorem**

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given collinear points A, B, C, and D arranged as shown, if $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Given: $\overline{AB} \cong \overline{CD}$
Prove: $\overline{AC} \cong \overline{BD}$

Flowchart proof:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{AB} \cong \overline{CD}$</td>
<td>Given</td>
</tr>
<tr>
<td>$\overline{AB} = \overline{CD}$</td>
<td>Def. of $\cong$ segs.</td>
</tr>
<tr>
<td>$\overline{BC} = \overline{BC}$</td>
<td>Reflex. Prop. of $=$</td>
</tr>
<tr>
<td>$\overline{AB} + \overline{BC} = \overline{AC}, \overline{BC} + \overline{CD} = \overline{BD}$</td>
<td>Seg. Add. Post.</td>
</tr>
<tr>
<td>$\overline{AB} + \overline{BC} = \overline{AC}, \overline{BC} + \overline{CD} = \overline{BD}$</td>
<td>Subst.</td>
</tr>
<tr>
<td>$\overline{AC} = \overline{BD}$</td>
<td>Def. of $\cong$ segs.</td>
</tr>
</tbody>
</table>

Two-column proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{AB} \cong \overline{CD}$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\overline{AB} = \overline{CD}$</td>
<td>2. Def. of $\cong$ segs.</td>
</tr>
<tr>
<td>3. $\overline{BC} = \overline{BC}$</td>
<td>3. Reflex. Prop. of $=$</td>
</tr>
<tr>
<td>4. $\overline{AB} + \overline{BC} = \overline{AC}, \overline{BC} + \overline{CD} = \overline{BD}$</td>
<td>4. Add. Prop. of $=$</td>
</tr>
<tr>
<td>5. $\overline{AB} + \overline{BC} = \overline{AC}, \overline{BC} + \overline{CD} = \overline{BD}$</td>
<td>5. Seg. Add. Post.</td>
</tr>
<tr>
<td>6. $\overline{AC} = \overline{BD}$</td>
<td>6. Subst.</td>
</tr>
<tr>
<td>7. $\overline{AC} \cong \overline{BD}$</td>
<td>7. Def. of $\cong$ segs.</td>
</tr>
</tbody>
</table>
1. Use the given flowchart proof to write a two-column proof.

Given: \( RS = UV, ST = TU \)
Prove: \( RT \cong TV \)

Flowchart proof:

```
Given: RS = UV, ST = TU

RS + ST = TU + UV
Add. Prop of =

RT = TV
Subst.

RT \cong TV
Def. of \cong segs.
```

2. Use the given two-column proof to write a flowchart proof.

Given: \( \angle 2 \cong \angle 4 \)
Prove: \( m\angle 1 = m\angle 3 \)

Two-column proof:

```
Statements | Reasons
--- | ---
1. \( \angle 2 \cong \angle 4 \) | 1. Given
2. \( \angle 1 \) and \( \angle 2 \) are supplementary.
   \( \angle 3 \) and \( \angle 4 \) are supplementary. | 2. Lin. Pair Thm.
3. \( \angle 1 \cong \angle 3 \) | 3. \( \cong \) Supps. Thm.
4. \( m\angle 1 = m\angle 3 \) | 4. Def. of \( = \)
A **paragraph proof** is a style of proof that presents the steps of the proof and their matching reasons as sentences in a paragraph. Although this style of proof is less formal than a two-column proof, you still must include every step.

### Theorems

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2-7-2 Vertical Angles Theorem</strong>&lt;br&gt;V Vertical angles are congruent.</td>
<td>∠A and ∠B are vertical angles.</td>
<td>∠A ≅ ∠B</td>
</tr>
<tr>
<td><strong>2-7-3</strong>&lt;br&gt;If two congruent angles are supplementary, then each angle is a right angle. (≅ supp. → rt. ∠)</td>
<td>∠1 ≅ ∠2&lt;br&gt;∠1 and ∠2 are supplementary.</td>
<td>∠1 and ∠2 are right angles.</td>
</tr>
</tbody>
</table>

#### Example 3

**Reading a Paragraph Proof**

Use the given paragraph proof to write a two-column proof of the Vertical Angles Theorem.

Given: ∠1 and ∠3 are vertical angles.  
Prove: ∠1 ≅ ∠3

**Paragraph proof:** ∠1 and ∠3 are vertical angles, so they are formed by intersecting lines. Therefore ∠1 and ∠2 are a linear pair, and ∠2 and ∠3 are a linear pair. By the Linear Pair Theorem, ∠1 and ∠2 are supplementary, and ∠2 and ∠3 are supplementary. So by the Congruent Supplements Theorem, ∠1 ≅ ∠3.

**Two-column proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ∠1 and ∠3 are vertical angles.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ∠1 and ∠3 are formed by intersecting lines.</td>
<td>2. Def. of vert. ∠</td>
</tr>
</tbody>
</table>
| 3. ∠1 and ∠2 are a linear pair.  
∠2 and ∠3 are a linear pair. | 3. Def. of lin. pair |
| 4. ∠1 and ∠2 are supplementary.  
∠2 and ∠3 are supplementary. | 4. Lin. Pair Thm. |
| 5. ∠1 ≅ ∠3 | 5. ≅ Supps. Thm. |

3. Use the given paragraph proof to write a two-column proof.

**Given:** ∠WXY is a right angle. ∠1 ≅ ∠3  
**Prove:** ∠1 and ∠2 are complementary.

**Paragraph proof:** Since ∠WXY is a right angle, m∠WXY = 90° by the definition of a right angle. By the Angle Addition Postulate, m∠WXY = m∠2 + m∠3. By substitution, m∠2 + m∠3 = 90°. Since ∠1 ≅ ∠3, m∠1 = m∠3 by the definition of congruent angles. Using substitution, m∠2 + m∠1 = 90°. Thus by the definition of complementary angles, ∠1 and ∠2 are complementary.
**EXAMPLE 4**

**Writing a Paragraph Proof**

Use the given two-column proof to write a paragraph proof of Theorem 2-7-3.

Given: ∠1 and ∠2 are supplementary. ∠1 ≅ ∠2

Prove: ∠1 and ∠2 are right angles.

Two-column proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ∠1 and ∠2 are supplementary. ∠1 ≅ ∠2</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. m∠1 + m∠2 = 180°</td>
<td>2. Def. of supp. ∠</td>
</tr>
<tr>
<td>3. m∠1 = m∠2</td>
<td>3. Def. of ≅ ∠ <em>Step 1</em></td>
</tr>
<tr>
<td>4. m∠1 + m∠1 = 180°</td>
<td>4. Subst. <em>Steps 2, 3</em></td>
</tr>
<tr>
<td>5. 2m∠1 = 180°</td>
<td>5. Simplification</td>
</tr>
<tr>
<td>6. m∠1 = 90°</td>
<td>6. Div. Prop. of =</td>
</tr>
<tr>
<td>7. m∠2 = 90°</td>
<td>7. Trans. Prop. of = <em>Steps 3, 6</em></td>
</tr>
<tr>
<td>8. ∠1 and ∠2 are right angles.</td>
<td>8. Def. of rt. ∠</td>
</tr>
</tbody>
</table>

Paragraph proof: ∠1 and ∠2 are supplementary, so m∠1 + m∠2 = 180° by the definition of supplementary angles. They are also congruent, so their measures are equal by the definition of congruent angles. By substitution, m∠1 + m∠1 = 180°, so m∠1 = 90° by the Division Property of Equality. Because m∠1 = m∠2, m∠2 = 90° by the Transitive Property of Equality. So both are right angles by the definition of a right angle.

4. Use the given two-column proof to write a paragraph proof.

Given: ∠1 ≅ ∠4

Prove: ∠2 ≅ ∠3

Two-column proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ∠1 ≅ ∠4</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ∠1 ≅ ∠2, ∠3 ≅ ∠4</td>
<td>2. Vert. ∆ Thm.</td>
</tr>
<tr>
<td>3. ∠2 ≅ ∠4</td>
<td>3. Trans. Prop. of ≅ <em>Steps 1, 2</em></td>
</tr>
<tr>
<td>4. ∠2 ≅ ∠3</td>
<td>4. Trans. Prop. of ≅ <em>Steps 2, 3</em></td>
</tr>
</tbody>
</table>
THINK AND DISCUSS
1. Explain why there might be more than one correct way to write a proof.
2. Describe the steps you take when writing a proof.
3. GET ORGANIZED
Copy and complete the graphic organizer.
In each box, describe the proof style in your own words.

GUIDED PRACTICE

Vocabulary  Apply the vocabulary from this lesson to answer each question.
1. In a ___ proof, the logical order is represented by arrows that connect each step.
   (flowchart or paragraph)

2. The steps and reasons of a ___ proof are written out in sentences.
   (flowchart or paragraph)

3. Use the given flowchart proof to write a two-column proof.
   Given: \( \angle 1 \cong \angle 2 \)
   Prove: \( \angle 1 \) and \( \angle 2 \) are right angles.
   Flowchart proof:

4. Use the given two-column proof to write a flowchart proof.
   Given: \( \angle 2 \) and \( \angle 4 \) are supplementary.
   Prove: \( m\angle 2 = m\angle 3 \)
   Two-column proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 2 ) and ( \angle 4 ) are supplementary.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 3 ) and ( \angle 4 ) are supplementary.</td>
<td>2. Lin. Pair Thm.</td>
</tr>
<tr>
<td>3. ( \angle 2 \cong \angle 3 )</td>
<td>3. ( \cong ) Supps. Thm.  Steps 1, 2</td>
</tr>
<tr>
<td>4. ( m\angle 2 = m\angle 3 )</td>
<td>4. Def. of ( \cong \Delta )</td>
</tr>
</tbody>
</table>
5. Use the given paragraph proof to write a two-column proof.

**Given:** \( \angle 2 \cong \angle 4 \)

**Prove:** \( \angle 1 \cong \angle 3 \)

**Paragraph proof:**

By the Vertical Angles Theorem, \( \angle 1 \cong \angle 2 \), and \( \angle 3 \cong \angle 4 \).

It is given that \( \angle 2 \cong \angle 4 \). By the Transitive Property of Congruence, \( \angle 1 \cong \angle 4 \), and thus \( \angle 1 \cong \angle 3 \).

6. Use the given two-column proof to write a paragraph proof.

**Given:** \( \overrightarrow{BD} \) bisects \( \angle ABC \).

**Prove:** \( \overrightarrow{BG} \) bisects \( \angle FBH \).

**Two-column proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overrightarrow{BD} ) bisects ( \angle ABC ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 1 \cong \angle 2 )</td>
<td>2. Def. of ( \angle ) bisector</td>
</tr>
<tr>
<td>3. ( \angle 1 \cong \angle 4 ), ( \angle 2 \cong \angle 3 )</td>
<td>3. Vert. ( \triangle ) Thm.</td>
</tr>
<tr>
<td>4. ( \angle 4 \cong \angle 2 )</td>
<td>4. Trans. Prop. of ( \cong ) Steps 2, 3</td>
</tr>
<tr>
<td>5. ( \angle 4 \cong \angle 3 )</td>
<td>5. Trans. Prop. of ( \cong ) Steps 3, 4</td>
</tr>
<tr>
<td>6. ( \overrightarrow{BG} ) bisects ( \angle FBH ).</td>
<td>6. Def. of ( \angle ) bisector</td>
</tr>
</tbody>
</table>

---

**PRACTICE AND PROBLEM SOLVING**

7. Use the given flowchart proof to write a two-column proof.

**Given:** \( B \) is the midpoint of \( AC \).

\[ AD = EC \]

**Prove:** \( DB = BE \)

**Flowchart proof:**

1. \( B \) is mdpt. of \( AC \).
2. \( AB = BC \)
3. Def. of mdpt.
4. \( AD + DB = AB \), \( BE + EC = BC \)
5. Seg. Add. Post

**Two-column proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( B ) is the midpoint of ( AC ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( AB = BC )</td>
<td>2. Def. of ( \angle ) bisector</td>
</tr>
<tr>
<td>4. Def. of supp. ( \angle )</td>
<td>4. Def. of supp. ( \angle )</td>
</tr>
<tr>
<td>5. ( DB = BE )</td>
<td>5. Subst. Steps 2, 4</td>
</tr>
</tbody>
</table>

8. Use the given two-column proof to write a flowchart proof.

**Given:** \( \angle 3 \) is a right angle.

**Prove:** \( \angle 4 \) is a right angle.

**Two-column proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 3 ) is a right angle.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle 3 = 90^\circ )</td>
<td>2. Def. of rt. ( \angle )</td>
</tr>
<tr>
<td>3. ( \angle 3 ) and ( \angle 4 ) are supplementary.</td>
<td>3. Lin. Pair Thm.</td>
</tr>
<tr>
<td>4. ( m\angle 3 + m\angle 4 = 180^\circ )</td>
<td>4. Def. of supp. ( \angle )</td>
</tr>
<tr>
<td>5. ( 90^\circ + m\angle 4 = 180^\circ )</td>
<td>5. Subst. Steps 2, 4</td>
</tr>
<tr>
<td>6. ( m\angle 4 = 90^\circ )</td>
<td>6. Subtr. Prop. of ( = )</td>
</tr>
<tr>
<td>7. ( \angle 4 ) is a right angle.</td>
<td>7. Def. of rt. ( \angle )</td>
</tr>
</tbody>
</table>
9. Use the given paragraph proof to write a two-column proof.

**Given:** ∠1 ≅ ∠4
**Prove:** ∠2 and ∠3 are supplementary.

**Paragraph proof:**

∠4 and ∠3 form a linear pair, so they are supplementary by the Linear Pair Theorem. Therefore, m∠4 + m∠3 = 180°. Also, ∠1 and ∠2 are vertical angles, so ∠1 ≅ ∠2 by the Vertical Angles Theorem. It is given that ∠1 ≅ ∠4. So by the Transitive Property of Congruence, ∠4 ≅ ∠2, and by the definition of congruent angles, m∠4 = m∠2. By substitution, m∠2 + m∠3 = 180°, so ∠2 and ∠3 are supplementary by the definition of supplementary angles.

10. Use the given two-column proof to write a paragraph proof.

**Given:** ∠1 and ∠2 are complementary.
**Prove:** ∠2 and ∠3 are complementary.

**Two-column proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ∠1 and ∠2 are complementary.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. m∠1 + m∠2 = 90°</td>
<td>2. Def. of comp. Δ</td>
</tr>
<tr>
<td>3. ∠1 ≅ ∠3</td>
<td>3. Vert. Δ Thm.</td>
</tr>
<tr>
<td>4. m∠1 = m∠3</td>
<td>4. Def. of ≅ Δ</td>
</tr>
<tr>
<td>5. m∠3 + m∠2 = 90°</td>
<td>5. Subst. Steps 2, 4</td>
</tr>
<tr>
<td>6. ∠2 and ∠3 are complementary.</td>
<td>6. Def. of comp. Δ</td>
</tr>
</tbody>
</table>

Find each measure and name the theorem that justifies your answer.

11. \( AB \) 22 cm
12. m∠2
13. m∠3

**Algebra** Find the value of each variable.

14. 17 in.
15. \( \frac{11y^2}{121} \)
16. \( \frac{(2x + 40)}{(5x + 16)} \)

17. **ERROR ANALYSIS** Below are two drawings for the given proof. Which is incorrect? Explain the error.

**Given:** \( \overline{AB} \cong \overline{BC} \)
**Prove:** ∠A ≅ ∠C

18. This problem will prepare you for the Multi-Step TAKS Prep on page 126. Rearrange the pieces to create a flowchart proof.
19. **Critical Thinking**  Two lines intersect, and one of the angles formed is a right angle. Explain why all four angles are congruent.

20. **Write About It**  Which style of proof do you find easiest to write? to read?

21. Which pair of angles in the diagram must be congruent?

   A) \( \angle 1 \) and \( \angle 5 \)  
   B) \( \angle 3 \) and \( \angle 4 \)  
   C) \( \angle 5 \) and \( \angle 8 \)  
   D) None of the above

22. What is the measure of \( \angle 2 \)?

   F) 38°  
   G) 52°  
   H) 128°  
   J) 142°

23. Which statement is NOT true if \( \angle 2 \) and \( \angle 6 \) are supplementary?

   A) \( m\angle 2 + m\angle 6 = 180^\circ \)  
   B) \( \angle 2 \) and \( \angle 3 \) are supplementary.  
   C) \( \angle 1 \) and \( \angle 6 \) are supplementary.  
   D) \( m\angle 1 + m\angle 4 = 180^\circ \)

24. **Textiles** Use the woven pattern to write a flowchart proof.

   Given: \( \angle 1 \equiv \angle 3 \)  
   Prove: \( m\angle 4 + m\angle 5 = m\angle 6 \)

25. Write a two-column proof.

   Given: \( \angle AOC \equiv \angle BOD \)  
   Prove: \( \angle AOB \equiv \angle COD \)

26. Write a paragraph proof.

   Given: \( \angle 2 \) and \( \angle 5 \) are right angles.  
   \( m\angle 1 + m\angle 2 + m\angle 3 = m\angle 4 + m\angle 5 + m\angle 6 \)  
   Prove: \( \angle 1 \equiv \angle 4 \)

27. **Multi-Step** Find the value of each variable and the measures of all four angles.

28. Solve each system of equations. Check your solution. *(Previous course)*

   \[ \begin{align*}
   y &= 2x + 14 \\
   y &= -6x + 18
   \end{align*} \]

   \[ \begin{align*}
   7x - y &= -33 \\
   3x + y &= -7
   \end{align*} \]

   \[ \begin{align*}
   2x + y &= 8 \\
   -x + 3y &= 10
   \end{align*} \]

29. Use a protractor to draw an angle with each of the following measures. *(Lesson 1-3)*

   31. 125°  
   32. 38°  
   33. 94°  
   34. 175°

30. For each conditional, write the converse and a biconditional statement. *(Lesson 2-4)*

   35. If a positive integer has more than two factors, then it is a composite number.

   36. If a quadrilateral is a trapezoid, then it has exactly one pair of parallel sides.
Mathematical Proof

Intersection Inspection  According to the U.S. Department of Transportation, it is ideal for two intersecting streets to form four 90° angles. If this is not possible, roadways should meet at an angle of 75° or greater for maximum safety and visibility.

1. Write a compound inequality to represent the range of measures an angle in an intersection should have.

2. Suppose that an angle in an intersection meets the guidelines specified by the U.S. Department of Transportation. Find the range of measures for the adjacent angle in the intersection.

The intersection of West Elm Street and Lamar Boulevard has a history of car accidents. The Southland neighborhood association is circulating a petition to have the city reconstruct the intersection. A surveyor measured the intersection, and one of the angles measures 145°.

3. Given that $m\angle 2 = 145°$, write a two-column proof to show that $m\angle 1$ and $m\angle 3$ are less than 75°.

4. Write a paragraph proof to justify the argument that the intersection of West Elm Street and Lamar Boulevard should be reconstructed.
Quiz for Lessons 2-5 Through 2-7

2-5 Algebraic Proof

Solve each equation. Write a justification for each step.
1. \( m - 8 = 13 \)
2. \( 4y - 1 = 27 \)
3. \( \frac{x}{3} = 2 \)

Identify the property that justifies each statement.
4. \( \angle XYZ = \angle PQR \), so \( \angle PQR = \angle XYZ \).
5. \( \overline{AB} \equiv \overline{AB} \)
6. \( \angle 4 \equiv \angle A \), and \( \angle A \equiv \angle 1 \). So \( \angle 4 \equiv \angle 1 \).
7. \( k = 7 \), and \( m = 7 \). So \( k = m \).

2-6 Geometric Proof

8. Fill in the blanks to complete the two-column proof.
   Given: \( \quad m\angle 1 + m\angle 3 = 180^\circ \)
   Prove: \( \angle 1 \equiv \angle 4 \)
   Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( m\angle 1 + m\angle 3 = 180^\circ )</td>
<td>1. a. _______?</td>
</tr>
<tr>
<td>2. b. _______?</td>
<td>2. Def. of supp. ( \angle )</td>
</tr>
<tr>
<td>3. ( \angle 3 ) and ( \angle 4 ) are supplementary.</td>
<td>3. Lin. Pair Thm.</td>
</tr>
<tr>
<td>4. ( \angle 3 \equiv \angle 3 )</td>
<td>4. c. _______?</td>
</tr>
<tr>
<td>5. d. _______?</td>
<td>5. ( \equiv ) Supps. Thm.</td>
</tr>
</tbody>
</table>

9. Use the given plan to write a two-column proof of the Symmetric Property of Congruence.
   Given: \( \overline{AB} \equiv \overline{EF} \)
   Prove: \( \overline{EF} \equiv \overline{AB} \)
   Plan: Use the definition of congruent segments to write \( \overline{AB} \equiv \overline{EF} \) as a statement of equality. Then use the Symmetric Property of Equality to show that \( EF = AB \). So \( \overline{EF} \equiv \overline{AB} \) by the definition of congruent segments.

2-7 Flowchart and Paragraph Proofs

Use the given two-column proof to write the following.
Given: \( \angle 1 \equiv \angle 3 \)
Prove: \( \angle 2 \equiv \angle 4 \)
Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 \equiv \angle 3 )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 1 \equiv \angle 2 ), ( \angle 3 \equiv \angle 4 )</td>
<td>2. Vert. ( \triangle ) Thm.</td>
</tr>
<tr>
<td>3. ( \angle 2 \equiv \angle 3 )</td>
<td>3. Trans. Prop. of ( \equiv )</td>
</tr>
<tr>
<td>4. ( \angle 2 \equiv \angle 4 )</td>
<td>4. Trans. Prop. of ( \equiv )</td>
</tr>
</tbody>
</table>

10. a flowchart proof
11. a paragraph proof
Symbolic logic is used by computer programmers, mathematicians, and philosophers to analyze the truth value of statements, independent of their actual meaning.

A **compound statement** is created by combining two or more statements. Suppose \( p \) and \( q \) each represent a statement. Two compound statements can be formed by combining \( p \) and \( q \): a *conjunction* and a *disjunction*.

### Compound Statements

<table>
<thead>
<tr>
<th>TERM</th>
<th>WORDS</th>
<th>SYMBOLS</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conjunction</td>
<td>A compound statement</td>
<td>( p \ AND \ q )</td>
<td>Pat is a band member</td>
</tr>
<tr>
<td></td>
<td>that uses the word</td>
<td>( p \land q )</td>
<td>AND Pat plays tennis.</td>
</tr>
<tr>
<td></td>
<td><em>and</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Disjunction</td>
<td>A compound statement</td>
<td>( p \ OR \ q )</td>
<td>Pat is a band member</td>
</tr>
<tr>
<td></td>
<td>that uses the word</td>
<td>( p \lor q )</td>
<td>OR Pat plays tennis.</td>
</tr>
<tr>
<td></td>
<td><em>or</em></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A conjunction is true only when all of its parts are true. A disjunction is true if any one of its parts is true.

### Example 1

**Analyzing Truth Values of Conjunctions and Disjunctions**

Use \( p \), \( q \), and \( r \) to find the truth value of each compound statement.

- \( p \): Washington, D.C., is the capital of the United States.
- \( q \): The day after Monday is Tuesday.
- \( r \): California is the largest state in the United States.

\[
\begin{align*}
\text{A} & \quad q \lor r \\
& \quad \text{Since } q \text{ is true,} \\
& \quad \text{the disjunction is true.} \\
\text{B} & \quad r \land p \\
& \quad \text{Since } r \text{ is false,} \\
& \quad \text{the conjunction is false.}
\end{align*}
\]

Use the information given above to find the truth value of each compound statement.

1a. \( r \lor p \)  
1b. \( p \land q \)

A table that lists all possible combinations of truth values for a statement is called a **truth table**. A truth table shows you the truth value of a compound statement, based on the possible truth values of its parts.
EXAMPLE 2 Constructing Truth Tables for Compound Statements

Construct a truth table for the compound statement \( \sim u \land (v \lor w) \).

Since \( u, v, \) and \( w \) can each be either true or false, the truth table will have \((2)(2)(2) = 8\) rows.

<table>
<thead>
<tr>
<th>( u )</th>
<th>( v )</th>
<th>( w )</th>
<th>( \sim u )</th>
<th>( v \lor w )</th>
<th>( \sim u \land (v \lor w) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
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<td>F</td>
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<td>T</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

2. Construct a truth table for the compound statement \( \sim u \land \sim v \).

Use \( p, q, \) and \( r \) to find the truth value of each compound statement.

\( p: \) The day after Friday is Sunday.
\( q: \frac{1}{2} = 0.5 \)
\( r: \) If \(-4x - 2 = 10\), then \( x = 3 \).

1. \( r \land q \)
2. \( r \lor p \)
3. \( p \lor r \)
4. \( q \land \sim q \)
5. \( \sim q \land q \)
6. \( q \lor r \)

Construct a truth table for each compound statement.

7. \( s \land \sim t \)
8. \( \sim u \lor t \)
9. \( \sim u \lor (s \land t) \)

Use a truth table to show that the two statements are logically equivalent.

10. \( p \rightarrow q; \sim q \rightarrow \sim p \)
11. \( q \rightarrow p; \sim p \rightarrow \sim q \)
12. A biconditional statement can be written as \((p \rightarrow q) \land (q \rightarrow p)\). Construct a truth table for this compound statement.
13. DeMorgan’s Laws state that \( \sim(p \land q) = \sim p \lor \sim q \) and that \( \sim(p \lor q) = \sim p \land \sim q \).
   a. Use truth tables to show that both statements are true.
   b. If you think of disjunction and conjunction as inverse operations, DeMorgan’s Laws are similar to which algebraic property?
14. The Law of Disjunctive Inference states that if \( p \lor q \) is true and \( p \) is false, then \( q \) must be true.
   a. Construct a truth table for \( p \lor q \).
   b. Use the truth table to explain why the Law of Disjunctive Inference is true.
Vocabulary

- biconditional statement ........................................... 96
- conclusion ................................................................. 81
- conditional statement ................................................. 81
- conjecture ................................................................. 74
- contrapositive ........................................................... 83
- converse ................................................................. 83
- counterexample ......................................................... 75
- deductive reasoning .................................................. 88
- definition ................................................................. 97
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Complete the sentences below with vocabulary words from the list above.

1. A statement you can prove and then use as a reason in later proofs is a(n) __?__.
2. __?__ is the process of using logic to draw conclusions from given facts, definitions, and properties.
3. A(n) __?__ is a case in which a conjecture is not true.
4. A statement you believe to be true based on inductive reasoning is called a(n) __?__.

2-1 Using Inductive Reasoning to Make Conjectures (pp. 74–79)

**Examples**

- Find the next item in the pattern below.
  
  The red square moves in the counterclockwise direction. The next figure is ___.

- Complete the conjecture “The sum of two odd numbers is __?__.”
  
  List some examples and look for a pattern.
  
  1 + 1 = 2  
  3 + 5 = 8  
  7 + 11 = 18
  
  The sum of two odd numbers is even.

- Show that the conjecture “For all non-zero integers, −x < x” is false by finding a counterexample.
  
  Pick positive and negative values for x and substitute to see if the conjecture holds.
  
  Let n = 3. Since −3 < 3, the conjecture holds.
  
  Let n = −5. Since −(−5) is 5 and 5 ≠ −5, the conjecture is false.
  
  n = −5 is a counterexample.

**Exercises**

Make a conjecture about each pattern. Write the next two items.

5. ___, ___, ___, ... ___, ___, ___, ...

6. ___, ___, ___, ...

7. ___, ___, ___, ...

Complete each conjecture.

8. The sum of an even number and an odd number is __?__.

9. The square of a natural number is __?__.

Determine if each conjecture is true. If not, write or draw a counterexample.

10. All whole numbers are natural numbers.

11. If C is the midpoint of AB, then AC ≅ BC.

12. If 2x + 3 = 15, then x = 6.

13. There are 28 days in February.

14. Draw a triangle. Construct the bisectors of each angle of the triangle. Make a conjecture about where the three angle bisectors intersect.
2-2 Conditional Statements (pp. 81–87)

**EXAMPLES**

- Write a conditional statement from the sentence “A rectangle has congruent diagonals.”
  
  If a figure is a rectangle, then it has congruent diagonals.

- Write the inverse, converse, and contrapositive of the conditional statement “If \( m\angle 1 = 35^\circ \), then \( \angle 1 \) is acute.” Find the truth value of each.
  
  Converse: If \( \angle 1 \) is acute, then \( m\angle 1 = 35^\circ \).
  Not all acute angles measure \( 35^\circ \), so this is false.
  
  Inverse: If \( m\angle 1 \neq 35^\circ \), then \( \angle 1 \) is not acute.
  You can draw an acute angle that does not measure \( 35^\circ \), so this is false.
  
  Contrapositive: If \( \angle 1 \) is not acute, then \( m\angle 1 \neq 35^\circ \). An angle that measures \( 35^\circ \) must be acute. So this statement is true.

2-3 Using Deductive Reasoning to Verify Conjectures (pp. 88–93)

**EXAMPLES**

- Determine if the conjecture is valid by the Law of Detachment or the Law of Syllogism.
  
  Given: If \( 5c = 8y \), then \( 2w = -15 \). If \( 5c = 8y \), then \( x = 17 \).
  
  Conjecture: If \( 2w = -15 \), then \( x = 17 \).
  Let \( p \) be \( 5c = 8y \), \( q \) be \( 2w = -15 \), and \( r \) be \( x = 17 \).
  
  Using symbols, the given information is written as \( p \rightarrow q \) and \( p \rightarrow r \). Neither the Law of Detachment nor the Law of Syllogism can be applied. The conjecture is not valid.

- Draw a conclusion from the given information.
  
  Given: If two points are distinct, then there is one line through them. \( A \) and \( B \) are distinct points.
  
  Let \( p \) be the hypothesis: two points are distinct.
  
  Let \( q \) be the conclusion: there is one line through the points.
  
  The statement “\( A \) and \( B \) are distinct points” matches the hypothesis, so you can conclude that there is one line through \( A \) and \( B \).

**EXERCISES**

- Write a conditional statement from each Venn diagram.
  
  15. [Weekdays Monday]
  16. [Fungi Lichen]

- Determine if each conditional is true. If false, give a counterexample.
  
  17. If two angles are adjacent, then they have a common ray.
  18. If you multiply two irrational numbers, the product is irrational.

- Write the converse, inverse, and contrapositive of each conditional statement. Find the truth value of each.
  
  19. If \( \angle X \) is a right angle, then \( m\angle X = 90^\circ \).
  20. If \( x \) is a whole number, then \( x = 2 \).

- Use the true statements below to determine whether each conclusion is true or false.
  
  Sue is a member of the swim team. When the team practices, Sue swims. The team begins practice when the pool opens. The pool opens at 8 A.M. on weekdays and at 12 noon on Saturday.
  
  21. The swim team practices on weekdays only.
  22. Sue swims on Saturdays.
  23. Swim team practice starts at the same time every day.

  Use the following information for Exercises 24–26.
  
  The expression \( 2.15 + 0.07x \) gives the cost of a long-distance phone call, where \( x \) is the number of minutes after the first minute.
  
  If possible, draw a conclusion from the given information. If not possible, explain why.
  
  24. The cost of Sara’s long-distance call is $2.57.
  25. Paulo makes a long-distance call that lasts ten minutes.
  26. Asa’s long-distance phone bill for the month is $19.05.
### Chapter 2 Geometric Reasoning

#### 2-4 Biconditional Statements and Definitions (pp. 96–101)

**Examples**

- For the conditional “If a number is divisible by 10, then it ends in 0”, write the converse and a biconditional statement.
  
  Converse: If a number ends in 0, then it is divisible by 10.
  
  Biconditional: A number is divisible by 10 if and only if it ends in 0.

- Determine if the biconditional “The sides of a triangle measure 3, 7, and 15 if and only if the perimeter is 25” is true. If false, give a counterexample.
  
  Conditional: If the sides of a triangle measure 3, 7, and 15, then the perimeter is 25. True.
  
  Converse: If the perimeter of a triangle is 25, then its sides measure 3, 7, and 15. False; a triangle with side lengths of 6, 10, and 9 also has a perimeter of 25. Therefore the biconditional is false.

#### 2-5 Algebraic Proof (pp. 104–109)

**Examples**

- Solve the equation $5x - 3 = -18$. Write a justification for each step.
  
  $\begin{align*}
  5x - 3 &= -18 & \text{Given} \\
  +3 & \quad +3 & \text{Add. Prop. of } = \\
  5x &= -15 & \text{Simplify} \\
  \frac{5x}{5} &= \frac{-15}{5} & \text{Div. Prop. of } = \\
  x &= -3 & \text{Simplify}
  \end{align*}$

- Write a justification for each step.
  
  $\begin{align*}
  RS &= ST & \text{Given} \\
  5x - 18 &= 4x & \text{Subst. Prop. of } = \\
  x - 18 &= 0 & \text{Subtr. Prop. of } = \\
  x &= 18 & \text{Add. Prop. of } =
  \end{align*}$

  Identify the property that justifies each statement.
  
  - $\angle X \cong \angle 2$, so $\angle 2 \cong \angle X$.
    
    Symmetric Property of Congruence
  
  - If $m\angle 2 = 180^\circ$ and $m\angle 3 = 180^\circ$, then $m\angle 2 = m\angle 3$.
    
    Transitive Property of Equality

**Exercises**

- Determine if a true biconditional can be written from each conditional statement. If not, give a counterexample.
  
  27. If $3 - \frac{2x}{5} = 2$, then $x = \frac{5}{2}$.
  
  28. If $x < 0$, then the value of $x^4$ is positive.
  
  29. If a segment has endpoints at $(1, 5)$ and $(-3, 1)$, then its midpoint is $(-1, 3)$.
  
  30. If the measure of one angle of a triangle is $90^\circ$, then the triangle is a right triangle.

- Complete each statement to form a true biconditional.
  
  31. Two angles are ___ if and only if the sum of their measures is $90^\circ$.
  
  32. $x^3 > 0$ if and only if $x$ is ___.
  
  33. Trey can travel 100 miles in less than 2 hours if and only if his average speed is ___.
  
  34. The area of a square is equal to $s^2$ if and only if the perimeter of the square is ___.

- For the conditional “If a number is divisible by 10, then it ends in 0”, write the converse and a biconditional statement.
  
  Converse: If a number ends in 0, then it is divisible by 10.
  
  Biconditional: A number is divisible by 10 if and only if it ends in 0.

- Determine if the biconditional “The sides of a triangle measure 3, 7, and 15 if and only if the perimeter is 25” is true. If false, give a counterexample.
  
  Conditional: If the sides of a triangle measure 3, 7, and 15, then the perimeter is 25. True.
  
  Converse: If the perimeter of a triangle is 25, then its sides measure 3, 7, and 15. False; a triangle with side lengths of 6, 10, and 9 also has a perimeter of 25. Therefore the biconditional is false.

- Solve the equation $5x - 3 = -18$. Write a justification for each step.
  
  $\begin{align*}
  5x - 3 &= -18 & \text{Given} \\
  +3 & \quad +3 & \text{Add. Prop. of } = \\
  5x &= -15 & \text{Simplify} \\
  \frac{5x}{5} &= \frac{-15}{5} & \text{Div. Prop. of } = \\
  x &= -3 & \text{Simplify}
  \end{align*}$

- Identify the property that justifies each statement.
  
  37. $a + b = a + b$
  
  38. If $\angle RST \cong \angle ABC$, then $\angle ABC \cong \angle RST$.
  
  39. $2x = 9$, and $y = 9$. So $2x = y$.

- Use the indicated property to complete each statement.
  
  40. Reflex. Prop. of $\cong$; figure $ABCD \cong ____$
  
  41. Sym. Prop. of $\cong$; If $m\angle 2 = m\angle 5$, then ____.
  
  42. Trans. Prop. of $\cong$; If $AB \cong CD$ and $AB \cong EF$, then ____.
  
  43. Kim borrowed money at an annual simple interest rate of 6% to buy a car. How much did she borrow if she paid $4200 in interest over the life of the 4-year loan? Solve the equation $I = Prt$ for $P$ and justify each step.
**EXAMPLES**

- Write a justification for each step, given that $m\angle 2 = 2m\angle 1$.
  1. $\angle 1$ and $\angle 2$ supp. Lin. Pair Thm.
  2. $m\angle 1 + m\angle 2 = 180°$ Def. of supp. $\triangle$
  3. $m\angle 2 = 2m\angle 1$ Given
  4. $m\angle 1 + 2m\angle 1 = 180°$ Subst. Steps 2, 3
  5. $3m\angle 1 = 180°$ Simplify
  6. $m\angle 1 = 60°$ Div. Prop. of =

- Use the given plan to write a two-column proof.
  **Given:** $\overline{AD}$ bisects $\angle BAC$.
  \[ \angle 1 \cong \angle 3 \]
  **Prove:** $\angle 2 \cong \angle 3$
  **Plan:** Use the definition of angle bisector to show that $\angle 1 \cong \angle 2$. Use the Transitive Property to conclude that $\angle 2 \cong \angle 3$.
  
  **Two-column proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{AD}$ bisects $\angle BAC$.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 1 \cong \angle 2$</td>
<td>2. Def. of $\angle$ bisector</td>
</tr>
<tr>
<td>3. $\angle 1 \cong \angle 3$</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. $\angle 2 \cong \angle 3$</td>
<td>4. Trans. Prop. of $\cong$</td>
</tr>
</tbody>
</table>

**EXERCISES**

44. Write a justification for each step, given that $\angle 1$ and $\angle 2$ are complementary, and $\angle 1 \cong \angle 3$.
  1. $\angle 1$ and $\angle 2$ comp.
  2. $m\angle 1 + m\angle 2 = 90°$ Def. of $\angle$ comp.
  3. $\angle 1 \cong \angle 3$
  4. $m\angle 1 = m\angle 3$
  5. $m\angle 3 + m\angle 2 = 90°$
  6. $\angle 3$ and $\angle 2$ comp.

45. Fill in the blanks to complete the two-column proof.
  **Given:** $\overline{TU} \cong \overline{UV}$
  **Prove:** $\overline{SU} + \overline{TU} = \overline{SV}$

  **Two-column proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{TU} \cong \overline{UV}$</td>
<td>1. a. $\cong$</td>
</tr>
<tr>
<td>2. $\overline{?}$</td>
<td>2. Def. of $\cong$ segs.</td>
</tr>
<tr>
<td>4. $\overline{ SU } + \overline{TU} = \overline{SV}$</td>
<td>4. d. $\cong$</td>
</tr>
</tbody>
</table>

46. Find the value of each variable.

47. Find the value of each variable and name the theorem that justifies your answer.

**Flowchart and Paragraph Proofs (pp. 118–125)**

**EXAMPLES**

- Use the two-column proof in the example for Lesson 2-6 above to write each of the following.
  - a flowchart proof

  Since $\overline{AD}$ bisects $\angle BAC$, $\angle 1 \cong \angle 2$ by the definition of angle bisector. It is given that $\angle 1 \cong \angle 3$. Therefore, $\angle 2 \cong \angle 3$ by the Transitive Property of Congruence.

- a paragraph proof

**EXERCISES**

Use the given plan to write each of the following.

**Given:** $\angle ADE$ and $\angle DAE$ are complementary.
$\angle ADE$ and $\angle BAC$ are complementary.

**Prove:** $\angle DAC \cong \angle BAE$

**Plan:** Use the Congruent Complements Theorem to show that $\angle DAE \cong \angle BAC$.
Since $\angle CAE \cong \angle CAE$, $\angle DAC \cong \angle BAE$ by the Common Angles Theorem.

48. a flowchart proof

49. a paragraph proof

50. Find the value of each variable and name the theorem that justifies your answer.
Find the next item in each pattern.

1. $$\square, \blacksquare, \blacksquare, \ldots$$
2. 405, 135, 45, 15, …

3. Complete the conjecture “The sum of two even numbers is ?.”

4. Show that the conjecture “All complementary angles are adjacent” is false by finding a counterexample.

5. Identify the hypothesis and conclusion of the conditional statement “The show is cancelled if it rains.”

6. Write a conditional statement from the sentence “Parallel lines do not intersect.”

Determine if each conditional is true. If false, give a counterexample.

7. If two lines intersect, then they form four right angles.

8. If a number is divisible by 10, then it is divisible by 5.

Use the conditional “If you live in the United States, then you live in Kentucky” for Items 9–11. Write the indicated type of statement and determine its truth value.

9. converse
10. inverse
11. contrapositive

12. Determine if the following conjecture is valid by the Law of Detachment.
   Given: If it is colder than 50°F, Tom wears a sweater. It is 46°F today.
   Conjecture: Tom is wearing a sweater.

13. Use the Law of Syllogism to draw a conclusion from the given information.
   Given: If a figure is a square, then it is a quadrilateral. If a figure is a quadrilateral, then it is a polygon. Figure $ABCD$ is a square.

14. Write the conditional statement and converse within the biconditional “Chad will work on Saturday if and only if he gets paid overtime.”

15. Determine if the biconditional “$B$ is the midpoint of $\overline{AC}$ iff $AB = BC$” is true. If false, give a counterexample.

Solve each equation. Write a justification for each step.

16. $8 - 5s = 1$
17. $0.4t + 3 = 1.6$
18. $38 = -3w + 2$

Identify the property that justifies each statement.

19. If $2x = y$ and $y = 7$, then $2x = 7$.
20. $m\angle DEF = m\angle DEF$
21. $\angle X \cong \angle P$, and $\angle P \cong \angle D$. So $\angle X \cong \angle D$.
22. If $\overline{ST} \cong \overline{XY}$, then $\overline{XY} \cong \overline{ST}$.

Use the given plan to write a proof in each format.

Given: $\angle AFB \cong \angle EFD$
Prove: $FB$ bisects $\angle AFC$.
Plan: Since vertical angles are congruent, $\angle EFD \cong \angle BFC$.
   Use the Transitive Property to conclude that $\angle AFB \cong \angle BFC$.
   Thus $FB$ bisects $\angle AFC$ by the definition of angle bisector.

23. two-column proof
24. paragraph proof
25. flowchart proof
FOCUS ON SAT MATHEMATICS SUBJECT TESTS

Some colleges require that you take the SAT Subject Tests. There are two math subject tests—Level 1 and Level 2. Take the Mathematics Subject Test Level 1 when you have completed three years of college-prep mathematics courses.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. In the figure below, $m\angle 1 = m\angle 2$. What is the value of $y$?

Note: Figure not drawn to scale.

(A) 10  (B) 30  (C) 40  (D) 50  (E) 60

2. The statement “I will cancel my appointment if and only if I have a conflict” is true. Which of the following can be concluded?

I. If I have a conflict, then I will cancel my appointment.

II. If I do not cancel my appointment, then I do not have a conflict.

III. If I cancel my appointment, then I have a conflict.

(A) I only  (B) II only  (C) III only  (D) I and III  (E) I, II, and III

3. What is the contrapositive of the statement “If it is raining, then the football team will win”?

(A) If it is not raining, then the football team will not win.

(B) If it is raining, then the football team will not win.

(C) If the football team wins, then it is raining.

(D) If the football team does not win, then it is not raining.

(E) If it is not raining, then the football team will win.

4. Given the points $D(1, 5)$ and $E(-2, 3)$, which conclusion is NOT valid?

(A) The midpoint of $DE$ is $\left(-\frac{1}{2}, 4\right)$.

(B) $D$ and $E$ are collinear.

(C) The distance between $D$ and $E$ is $\sqrt{5}$.

(D) $\overline{DE} \cong \overline{ED}$

(E) $D$ and $E$ are distinct points.

5. For all integers $x$, what conclusion can be drawn about the value of the expression $\frac{x^2}{2}$?

(A) The value is negative.

(B) The value is not negative.

(C) The value is even.

(D) The value is odd.

(E) The value is not a whole number.
Gridded Response: Record Your Answer

When responding to a gridded-response test item, you must fill out the grid on your answer sheet correctly, or the item will be marked as incorrect.

**Example 1**

Gridded Response: Solve the equation $1258 - 2(3x - 72) = -80$.

The value of $x$ is 247.

- Using a pencil, write your answer in the answer boxes at the top of the grid.
- Put only one digit in each box. The decimal point has a designated column.
- Do not leave a blank box in the middle of an answer.
- For each digit, shade the bubble that is in the same column as the digit in the answer box.

**Example 2**

Gridded Response: The perimeter of a rectangle is 90 in. The width of the rectangle is 18 in. Find the length of the rectangle in feet.

The length of the rectangle is 27 inches, but the problem asks for the measurement in feet.

27 inches = $\frac{9}{4}$, or 2.25, feet

- Fractions and mixed numbers cannot be gridded, so you must grid the answer as 2.25.
- Using a pencil, write your answer in the answer boxes at the top of the grid.
- Put only one digit in each box. The decimal point has a designated column.
- Do not leave a blank box in the middle of an answer.
- For each digit, shade the bubble that is in the same column as the digit in the answer box.
You cannot grid a negative number in a gridded-response item because the grid does not include the negative sign (−). So if you get a negative answer to a test item, rework the problem. You probably made a math error.

Read each statement and answer the questions that follow.

**Sample A**
The correct answer to a test item is 1.6. A student gridded this answer as shown.

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</tbody>
</table>

1. What error did the student make when filling out the grid?
2. Another student got an answer of −1.6. Explain why the student knew this answer was wrong.

**Sample B**
The perimeter of a triangle is $2\frac{3}{4}$ feet. A student gridded this answer as shown.

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3. What error did the student make when filling out the grid?
4. Explain how to correctly grid the answer.

**Sample C**
The length of a segment is $897\frac{2}{5}$ units. A student gridded this answer as shown.

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5. What answer does the grid show?
6. Explain why you cannot grid a fraction or a mixed number.
7. Write the answer $897\frac{2}{5}$ in a form that could be entered in the grid correctly.
8. Another student got an answer of 10,216.5 units. Explain why the student knew this answer was wrong.

**Sample D**
The measure of an angle is 48.9°. A student gridded this answer as shown.

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</table>

9. What answer does the grid show?
10. What error did the student make when filling out the grid?
11. Explain how to correctly grid the answer.
Multiple Choice

Use the figure below for Items 1 and 2. In the figure, \( \overline{DB} \) bisects \( \angle ADC \).

1. Which best describes the intersection of \( \angle ADB \) and \( \angle BDC \)?
   - A) Exactly one ray
   - B) Exactly one point
   - C) Exactly one angle
   - D) Exactly one segment

2. Which expression is equal to the measure of \( \angle ADC \)?
   - F) \( 2(\angle ADB) \)
   - G) \( 90^\circ - \angle BDC \)
   - H) \( 180^\circ - 2(\angle ADC) \)
   - J) \( \angle BDC - \angle ADB \)

3. What is the inverse of the statement, “If a polygon has 8 sides, then it is an octagon”?
   - A) If a polygon is an octagon, then it has 8 sides.
   - B) If a polygon is not an octagon, then it does not have 8 sides.
   - C) If an octagon has 8 sides, then it is a polygon.
   - D) If a polygon does not have 8 sides, then it is not an octagon.

4. Lily conjectures that if a number is divisible by 15, then it is also divisible by 9. Which of the following is a counterexample?
   - F) 45
   - G) 50
   - H) 60
   - J) 72

5. A diagonal of a polygon connects nonconsecutive vertices. The table shows the number of diagonals in a polygon with \( n \) sides.

<table>
<thead>
<tr>
<th>Number of Sides</th>
<th>Number of Diagonals</th>
</tr>
</thead>
<tbody>
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<tr>
<td>7</td>
<td>14</td>
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</tbody>
</table>

If the pattern continues, how many diagonals does a polygon with 8 sides have?
   - A) 17
   - B) 19
   - C) 20
   - D) 21

6. Which type of transformation maps figure \( LMNP \) onto figure \( L'M'N'P' \)?
   - F) Reflection
   - G) Rotation
   - H) Translation
   - J) None of these

7. Miyoko went jogging on July 25, July 28, July 31, and August 3. If this pattern continues, when will Miyoko go jogging next?
   - A) August 5
   - B) August 6
   - C) August 7
   - D) August 8

8. Congruent segments have equal measures. A segment bisector divides a segment into two congruent segments. \( XY \) intersects \( DE \) at \( X \) and bisects \( DE \). Which conjecture is valid?
   - F) \( \angle YXD = \angle YXE \)
   - G) \( Y \) is between \( D \) and \( E \).
   - H) \( DX = XE \)
   - J) \( DE = YE \)
9. Which statement is true by the Symmetric Property of Congruence?
   A $ST \equiv TS$
   B $15 + MN = MN + 15$
   C If $\angle P \equiv \angle Q$, then $\angle Q \equiv \angle P$.
   D If $\angle D \equiv \angle E$ and $\angle E \equiv \angle F$, then $\angle D \equiv \angle F$.

To find a counterexample for a biconditional statement, write the conditional statement and converse it contains. Then try to find a counterexample for one of these statements.

10. Which is a counterexample for the following biconditional statement?
   A A pair of angles is supplementary if and only if the angles form a linear pair.
   F The measures of supplementary angles add to 180°.
   G A linear pair of angles is supplementary.
   H Complementary angles do not form a linear pair.
   J Two supplementary angles are not adjacent.

11. K is between J and L. The distance between J and K is 3.5 times the distance between K and L. If $JK = 14$, what is $JL$?
   A $10.5$
   B $18$
   C $24.5$
   D $49$

12. What is the length of the segment connecting the points $(-7, -5)$ and $(5, -2)$?
   F $\sqrt{13}$
   G $\sqrt{53}$
   H $3\sqrt{17}$
   J $\sqrt{193}$

Gridded Response

13. A segment has an endpoint at $(5, -2)$. The midpoint of the segment is $(2, 2)$. What is the length of the segment?

14. $\angle P$ measures $30°$ more than the measure of its supplement. What is the measure of $\angle P$ in degrees?

15. The perimeter of a square field is 1.6 kilometers. What is the area of the field in square kilometers?

16. Solve the equation $2(AB) + 16 = 24$ to find the length of segment $AB$. Write a justification for each step.

17. Use the given two-column proof to write a flowchart proof.

```
Given: DE $\equiv$ FH
Prove: DE = FG + GH
```

Two-column proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $DE \equiv FH$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $DE = FH$</td>
<td>2. Def. of $\equiv$ segs.</td>
</tr>
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</table>

18. Consider the following conditional statement.
   If two angles are complementary, then the angles are acute.
   a. Determine if the conditional is true or false. If false, give a counterexample.
   b. Write the converse of the conditional statement.
   c. Determine whether the converse is true or false. If false, give a counterexample.

19. The figure below shows the intersection of two lines.

```
A   E
   C   B
   D
```

a. Name the linear pairs of angles in the figure. What conclusion can you make about each pair? Explain your reasoning.

b. Name the pairs of vertical angles in the figure. What conclusion can you make about each pair? Explain your reasoning.

c. Suppose $m\angle EBD = 90°$. What are the measures of the other angles in the figure? Write a two-column proof to support your answer.
The Freescale Marathon

Every February, runners take to the streets of Austin to participate in a 26-mile marathon. The course travels mostly downhill, with a net drop in elevation of more than 400 feet from beginning to end. But don't be fooled; many former participants strongly recommend hill training for this course!

Choose one or more strategies to solve each problem.

1. During the marathon, a runner maintains a steady pace and completes the first 2.6 miles in 20 minutes. After 1 hour 20 minutes, she has completed 10.4 miles. Make a conjecture about the runner's average speed in miles per hour. How long do you expect her to take to complete the marathon?

2. The course features fully equipped aid stations with medical support for the complete eight-hour duration of the race. From mile 2 to mile 6, these stations are located every other mile. After that, they are located every mile to the finish. Portable toilets are available at each mile marker and at the end of the course. At how many points are there both an aid station and portable toilets?

For 3, use the map.

3. The course includes a straight section along Forty-fifth Street from Shoal Creek Boulevard to Duval Street. The distance from Guadalupe Street to Duval is twice the distance from Burnet Road to Lamar Boulevard. The distance from Lamar to Guadalupe is 210 feet greater than the distance from Shoal Creek to Burnet. What is the distance from Guadalupe to Duval?
Show Caves

The region between San Antonio and Austin, known as the Texas Hill Country, is home to six of the seven show caves in the state. A show cave is a cave developed for public use, typically with amenities such as lighting and groomed trails. The seventh show cave in Texas, the Caverns of Sonora, is internationally recognized as one of the most beautiful in the world.

Choose one or more strategies to solve each problem.

1. Jared took a tour at least $\frac{5}{6}$ mi long and saw a cave that is less than 10,000 ft in length. Which caves might Jared have visited?

2. A travel brochure includes the following statements about Texas show caves. Determine whether each statement is true or false. If false, explain why.
   a. If you tour a cave that is more than 100 ft deep, then you'll see a cave that is more than 8000 ft in length.
   b. If you haven't been to the Caverns of Sonora, then you haven't seen a cave that is at least 15,000 ft long.
   c. If you don't want to walk more than a mile, but you want to see a cave with a depth of at least 150 ft, then you should visit Natural Bridge Caverns.

3. Inner Space Cavern has a second tour, which Ingrid completes in 18 min and 45 s. If Ingrid walks 12,672 ft in 1 h, what is the length of the second tour?

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<th>Approximate Depth (ft)</th>
<th>Tour Length (mi)</th>
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