Vocabulary
Match each term on the left with a definition on the right.
1. acute angle         A. a statement that is accepted as true without proof
2. congruent segments  B. an angle that measures greater than 90° and less than 180°
3. obtuse angle        C. a statement that you can prove
4. postulate           D. segments that have the same length
5. triangle            E. a three-sided polygon
                      F. an angle that measures greater than 0° and less than 90°

Measure Angles
Use a protractor to measure each angle.
6. 

7. 

Use a protractor to draw an angle with each of the following measures.
8. 20°  9. 63°
10. 105°  11. 158°

Solve Equations with Fractions
Solve.
12. \( \frac{9}{2}x + 7 = 25 \)
13. \( 3x - \frac{2}{3} = \frac{4}{3} \)
14. \( x - \frac{1}{5} = \frac{12}{5} \)
15. \( 2y = 5y - \frac{21}{2} \)

Connect Words and Algebra
Write an equation for each statement.
16. Tanya’s age \( t \) is three times Martin’s age \( m \).
17. Twice the length of a segment \( x \) is 9 ft.
18. The sum of 53° and twice an angle measure \( y \) is 90°.
19. The price of a radio \( r \) is $25 less than the price of a CD player \( p \).
20. Half the amount of liquid \( j \) in a jar is 5 oz more than the amount of liquid \( b \) in a bowl.
Key Vocabulary/Vocabulario

- acute triangle: triángulo acutángulo
- congruent polygons: polígonos congruentes
- corollary: corolario
- equilateral triangle: triángulo equilátero
- exterior angle: ángulo externo
- interior angle: ángulo interno
- isosceles triangle: triángulo isósceles
- obtuse triangle: triángulo obtusángulo
- right triangle: triángulo rectángulo
- scalene triangle: triángulo escaleno

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The Latin word *acutus* means “pointed” or “sharp.” Draw a triangle that looks pointed or sharp. Do you think this is an acute triangle?

2. Consider the everyday meaning of the word exterior. Where do you think an exterior angle of a triangle is located?

3. You already know the definition of an obtuse angle. Use this meaning to make a conjecture about an obtuse triangle.

Geometry TEKS

<table>
<thead>
<tr>
<th>Knowledge and skills</th>
<th>Les. 4-1</th>
<th>Les. 4-2</th>
<th>Les. 4-3</th>
<th>Les. 4-4</th>
<th>Les. 4-5</th>
<th>Les. 4-6</th>
<th>Les. 4-7</th>
<th>Les. 4-8</th>
<th>Ext.</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.1.A Geometric structure* develop an awareness of the structure of a mathematical system ...</td>
<td>⭐️ ⭐️ ⭐️</td>
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<td>G.2.A Geometric structure* use constructions to explore attributes of geometric figures and to make conjectures ...</td>
<td>⭐️ ⭐️ ⭐️</td>
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<tr>
<td>G.2.B Geometric structure* make conjectures about angles, lines, polygons ... and determine validity of the conjectures, choosing from a variety of approaches ...</td>
<td>⭐️ ⭐️</td>
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<tr>
<td>G.5.A Geometric patterns* use ... geometric patterns to develop algebraic expressions representing geometric properties</td>
<td>⭐️</td>
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<tr>
<td>G.7.A Dimensionality and the geometry of location* use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures</td>
<td>⭐️</td>
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<tr>
<td>G.9.B Congruence and the geometry of size* formulate and test conjectures about the properties and attributes of polygons ... based on explorations ...</td>
<td>⭐️ ⭐️ ⭐️ ⭐️</td>
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<tr>
<td>G.10.B Congruence and the geometry of size* justify and apply triangle congruence relationships</td>
<td>⭐️ ⭐️ ⭐️ ⭐️ ⭐️ ⭐️</td>
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</tbody>
</table>

* Knowledge and skills are written out completely on pages TX28–TX35.
Reading and Writing

**Reading Strategy: Read Geometry Symbols**

In Geometry we often use symbols to communicate information. When studying each lesson, read both the symbols and the words slowly and carefully. Reading aloud can sometimes help you translate symbols into words.

Throughout this course, you will use these symbols and combinations of these symbols to represent various geometric statements.

<table>
<thead>
<tr>
<th>Symbol Combinations</th>
<th>Translated into Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ST \parallel UV$</td>
<td>Line $ST$ is parallel to line $UV$.</td>
</tr>
<tr>
<td>$BC \perp GH$</td>
<td>Segment $BC$ is perpendicular to segment $GH$.</td>
</tr>
<tr>
<td>$p \rightarrow q$</td>
<td>If $p$, then $q$.</td>
</tr>
<tr>
<td>$m\angle QRS = 45^\circ$</td>
<td>The measure of angle $QRS$ is 45 degrees.</td>
</tr>
<tr>
<td>$\angle CDE \cong \angle LMN$</td>
<td>Angle $CDE$ is congruent to angle $LMN$.</td>
</tr>
</tbody>
</table>

**Try This**

**Rewrite each statement using symbols.**

1. The absolute value of 2 times pi
2. The measure of angle 2 is 125 degrees.
3. Segment $XY$ is perpendicular to line $BC$.
4. If not $p$, then not $q$.

**Translate the symbols into words.**

5. $m\angle FGH = m\angle VWX$
6. $ZA \parallel TU$
7. $\sim p \rightarrow q$
8. $ST$ bisects $\angle TSU$. 

**Triangle Congruence**
Objectives
Classify triangles by their angle measures and side lengths.
Use triangle classification to find angle measures and side lengths.

Vocabulary
acute triangle
equiangular triangle
right triangle
obtuse triangle
equilateral triangle
isosceles triangle
scalene triangle

Who uses this?
Manufacturers use properties of triangles to calculate the amount of material needed to make triangular objects. (See Example 4.)

A triangle is a steel percussion instrument in the shape of an equilateral triangle. Different-sized triangles produce different musical notes when struck with a metal rod.

Recall that a triangle (\( \triangle \)) is a polygon with three sides. Triangles can be classified in two ways: by their angle measures or by their side lengths.

A \( \triangle \) is an equilateral triangle by definition.

Example 1
Classify each triangle by its angle measures.

A \( \triangle EHG \)
\( \angle EHG \) is a right angle. So \( \triangle EHG \) is a right triangle.

B \( \triangle EFH \)
\( \angle EFH \) and \( \angle HFG \) form a linear pair, so they are supplementary. Therefore \( m\angle EFH + m\angle HFG = 180^\circ \). By substitution,

\[ m\angle EFH + 60^\circ = 180^\circ. \]
So \( m\angle EFH = 120^\circ \). \( \triangle EFH \) is an obtuse triangle by definition.

1. Use the diagram to classify \( \triangle FHG \) by its angle measures.
Classifying Triangles by Side Lengths

Example 2

Classify each triangle by its side lengths.

A. ΔABC
   From the figure, \( AB \cong AC \). So \( AC = 15 \), and ΔABC is equilateral.

B. ΔABD
   By the Segment Addition Postulate, 
   \( BD = BC + CD = 15 + 5 = 20 \). Since no sides are congruent, ΔABD is scalene.

Example 3

Using Triangle Classification

Find the side lengths of the triangle.

Step 1 Find the value of \( \ell \).
   - \( JK \cong KL \)  
   - \( JK = KL \)
   - \( (4x - 1.3) = (x + 3.2) \)  
     Substitute \((4x - 13)\) for \( JK \) and \((x + 3.2)\) for \( KL \).
   - \( 3x = 4.5 \)
   - \( x = 1.5 \)

Step 2 Substitute 1.5 into the expressions to find the side lengths.
   - \( JK = 4x - 1.3 \)
     \( = 4(1.5) - 1.3 = 4.7 \)
   - \( KL = x + 3.2 \)
     \( = 1.5 + 3.2 = 4.7 \)
   - \( JL = 5x - 0.2 \)
     \( = 5(1.5) - 0.2 = 7.3 \)

3. Find the side lengths of equilateral ΔFGH.
Music Application

A manufacturer produces musical triangles by bending pieces of steel into the shape of an equilateral triangle. The triangles are available in side lengths of 4 inches, 7 inches, and 10 inches. How many 4-inch triangles can the manufacturer produce from a 100 inch piece of steel?

The amount of steel needed to make one triangle is equal to the perimeter $P$ of the equilateral triangle.

$$P = 3(4) = 12 \text{ in.}$$

To find the number of triangles that can be made from 100 inches of steel, divide 100 by the amount of steel needed for one triangle.

$$100 \div 12 = 8 \frac{2}{3} \text{ triangles}$$

There is not enough steel to complete a ninth triangle. So the manufacturer can make 8 triangles from a 100 in. piece of steel.

Each measure is the side length of an equilateral triangle. Determine how many triangles can be formed from a 100 in. piece of steel.

4a. 7 in.  
4b. 10 in.

Think and Discuss

1. For $\triangle DEF$, name the three pairs of consecutive sides and the vertex formed by each.

2. Sketch an example of an obtuse isosceles triangle, or explain why it is not possible to do so.

3. Is every acute triangle equiangular? Explain and support your answer with a sketch.

4. Use the Pythagorean Theorem to explain why you cannot draw an equilateral right triangle.

5. GET ORGANIZED Copy and complete the graphic organizer. In each box, describe each type of triangle.
**4-1 Classifying Triangles**

### GUIDED PRACTICE

**Vocabulary** Apply the vocabulary from this lesson to answer each question.

1. In $\triangle JKL$, $JK$, $KL$, and $JL$ are equal. How does this help you classify $\triangle JKL$ by its side lengths?

2. $\triangle XYZ$ is an obtuse triangle. What can you say about the types of angles in $\triangle XYZ$?

#### Classify each triangle by its angle measures.

3. $\triangle DBC$  
4. $\triangle ABD$  
5. $\triangle ADC$

#### Classify each triangle by its side lengths.

6. $\triangle EGH$  
7. $\triangle EFH$  
8. $\triangle HFG$

#### Multi-Step Find the side lengths of each triangle.

9. $\triangle$  
10. $\triangle$

11. **Crafts** A jeweler creates triangular earrings by bending pieces of silver wire. Each earring is an isosceles triangle with the dimensions shown. How many earrings can be made from a piece of wire that is 50 cm long?

### PRACTICE AND PROBLEM SOLVING

Classify each triangle by its angle measures.

12. $\triangle BEA$  
13. $\triangle DBC$  
14. $\triangle ABC$

Classify each triangle by its side lengths.

15. $\triangle PST$  
16. $\triangle RSP$  
17. $\triangle RPT$

#### Multi-Step Find the side lengths of each triangle.

18. $\triangle$  
19. $\triangle$

20. Draw a triangle large enough to measure. Label the vertices $X$, $Y$, and $Z$.
   
a. Name the three sides and three angles of the triangle.
   
b. Use a ruler and protractor to classify the triangle by its side lengths and angle measures.
Carpentry  Use the following information for Exercises 21 and 22.
A manufacturer makes trusses, or triangular supports, for the roofs of houses. Each truss is the shape of an isosceles triangle in which $PQ \cong PR$. The length of the base $QR$ is $\frac{4}{3}$ the length of each of the congruent sides.

21. The perimeter of each truss is 60 ft.
   Find each side length.

22. How many trusses can the manufacturer make from 150 feet of lumber?

Draw an example of each type of triangle or explain why it is not possible.

23. isosceles right
24. equiangular obtuse
25. scalene right
26. equilateral acute
27. scalene equiangular
28. isosceles acute

29. An equilateral triangle has a perimeter of 105 in.
   What is the length of each side of the triangle?

Classify each triangle by its angles and sides.

30. $\triangle ABC$
31. $\triangle ACD$

32. An isosceles triangle has a perimeter of 34 cm. The congruent sides measure $(4x - 1)$ cm. The length of the third side is $x$ cm. What is the value of $x$?

Architecture  The base of the Flatiron Building is a triangle bordered by three streets: Broadway, Fifth Avenue, and East Twenty-second Street. The Fifth Avenue side is 1 ft shorter than twice the East Twenty-second Street side. The East Twenty-second Street side is 8 ft shorter than half the Broadway side. The Broadway side is 190 ft.

a. Find the two unknown side lengths.
b. Classify the triangle by its side lengths.


Tell whether each statement is sometimes, always, or never true. Support your answer with a sketch.

35. An acute triangle is a scalene triangle.
36. A scalene triangle is an obtuse triangle.
37. An equiangular triangle is an isosceles triangle.
38. Write About It  Write a formula for the side length $s$ of an equilateral triangle, given the perimeter $P$. Explain how you derived the formula.
39. Construction  Use the method for constructing congruent segments to construct an equilateral triangle.

40. This problem will prepare you for the Multi-Step TAKS Prep on page 238.
Marc folded a rectangular sheet of paper, $ABCD$, in half along $EF$. He folded the resulting square diagonally and then unfolded the paper to create the creases shown.

a. Use the Pythagorean Theorem to find $DE$ and $CE$.
b. What is the $m\angle DEC$?
c. Classify $\triangle DEC$ by its side lengths and by its angle measures.
41. What is the side length of an equilateral triangle with a perimeter of $36\frac{2}{3}$ inches?
- A $36\frac{2}{3}$ inches
- B $18\frac{1}{3}$ inches
- C $12\frac{1}{3}$ inches
- D $12\frac{2}{9}$ inches

42. The vertices of $\triangle RST$ are $R(3, 2)$, $S(-2, 3)$, and $T(-2, 1)$. Which of these best describes $\triangle RST$?
- F Isosceles
- G Scalene
- H Equilateral
- J Right

43. Which of the following is NOT a correct classification of $\triangle LMN$?
- A Acute
- B Equiangular
- C Isosceles
- D Right

44. Gridded Response $\triangle ABC$ is isosceles, and $AB \cong AC$. $AB = \left(\frac{1}{2}x + \frac{1}{4}\right)$, and $BC = \left(\frac{5}{2} - x\right)$. What is the perimeter of $\triangle ABC$?

CHALLENGE AND EXTEND

45. A triangle has vertices with coordinates $(0, 0)$, $(a, 0)$, and $(0, a)$, where $a \neq 0$. Classify the triangle in two different ways. Explain your answer.

46. Write a two-column proof.
   Given: $\triangle ABC$ is equiangular.
   $EF \parallel AC$
   Prove: $\triangle EFB$ is equiangular.

47. Two sides of an equilateral triangle measure $(y + 10)$ units and $(y^2 - 2)$ units. If the perimeter of the triangle is 21 units, what is the value of $y$?

48. Multi-Step The average length of the sides of $\triangle PQR$ is 24. How much longer then the average is the longest side?

SPIRAL REVIEW

Name the parent function of each function. (Previous course)

49. $y = 5x^2 + 4$
50. $2y = 3x + 4$
51. $y = 2(x - 8)^2 + 6$

Determine if each biconditional is true. If false, give a counterexample. (Lesson 2-4)

52. Two lines are parallel if and only if they do not intersect.
53. A triangle is equiangular if and only if it has three congruent angles.
54. A number is a multiple of 20 if and only if the number ends in a 0.

Determine whether each line is parallel to, is perpendicular to, or coincides with $y = 4x$. (Lesson 3-6)

55. $y = 4x + 2$
56. $4y = -x + 8$
57. $\frac{1}{2}y = 2x$
58. $-2y = \frac{1}{2}x$
Develop the Triangle Sum Theorem

In this lab, you will use patty paper to discover a relationship between the measures of the interior angles of a triangle.

**Activity**

1. Draw and label ΔABC on a sheet of notebook paper.

2. On patty paper draw a line ℓ and label a point P on the line.

3. Place the patty paper on top of the triangle you drew. Align the papers so that AB is on line ℓ and P and B coincide. Trace ∠B. Rotate the triangle and trace ∠C adjacent to ∠B. Rotate the triangle again and trace ∠A adjacent to ∠C. The diagram shows your final step.

**Try This**

1. What do you notice about the three angles of the triangle that you traced?

2. Repeat the activity two more times using two different triangles. Do you get the same results each time?

3. Write an equation describing the relationship among the measures of the angles of ΔABC.

4. Use inductive reasoning to write a conjecture about the sum of the measures of the angles of a triangle.
Who uses this?
Surveyors use triangles to make measurements and create boundaries. (See Example 1.)

Triangulation is a method used in surveying. Land is divided into adjacent triangles. By measuring the sides and angles of one triangle and applying properties of triangles, surveyors can gather information about adjacent triangles.

The sum of the angle measures of a triangle is 180°.

\[ m\angle A + m\angle B + m\angle C = 180° \]

**Theorem 4-2-1  Triangle Sum Theorem**

The proof of the Triangle Sum Theorem uses an auxiliary line. An auxiliary line is a line that is added to a figure to aid in a proof.

**Proof**

**Triangle Sum Theorem**

Given: \( \triangle ABC \)
Prove: \( m\angle 1 + m\angle 2 + m\angle 3 = 180° \)

Proof:

1. Draw \( \ell \parallel \overline{AC} \) through \( B \).
   - Parallel Post.
2. \( \angle 1 \cong \angle 4 \) and \( \angle 3 \cong \angle 5 \) by the Alternate Interior Angles Theorem.
3. \( m\angle 1 = m\angle 4 \) and \( m\angle 3 = m\angle 5 \) by the Definition of \( \cong \) Angle.
4. \( m\angle 4 + m\angle 2 + m\angle 5 = 180° \) by the Angle Addition Postulate and the Definition of Straight Angle.
5. \( m\angle 1 + m\angle 2 + m\angle 3 = 180° \) by Substitution.

**Caution!**
Whenever you draw an auxiliary line, you must be able to justify its existence. Give this as the reason: Through any two points there is exactly one line.
EXAMPLE 1

Surveying Application

The map of France commonly used in the 1600s was significantly revised as a result of a triangulation land survey. The diagram shows part of the survey map. Use the diagram to find the indicated angle measures.

A \( m\angle NKM \)

\[ m\angle KMN + m\angle MNK + m\angle NKM = 180° \]

\[ 88 + 48 + m\angle NKM = 180 \]

\[ 136 + m\angle NKM = 180 \]

\[ m\angle NKM = 44° \]

B \( m\angle JLK \)

Step 1 Find \( m\angle JKL \).

\[ m\angle NKM + m\angle MKJ + m\angle JKL = 180° \]

\[ 44 + 104 + m\angle JKL = 180 \]

\[ 148 + m\angle JKL = 180 \]

\[ m\angle JKL = 32° \]

Step 2 Use substitution and then solve for \( m\angle JLK \).

\[ m\angle JLK + m\angle JKL + m\angle KJL = 180° \]

\[ m\angle JLK + 32 + 70 = 180 \]

\[ m\angle JLK + 102 = 180 \]

\[ m\angle JLK = 78° \]

1. Use the diagram to find \( m\angle MJK \).

A corollary is a theorem whose proof follows directly from another theorem. Here are two corollaries to the Triangle Sum Theorem.

**Corollaries**

<table>
<thead>
<tr>
<th>COROLLARY</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-2-2</td>
<td>[Diagram of triangle with angles D and E]</td>
<td>( \angle D ) and ( \angle E ) are complementary. ( m\angle D + m\angle E = 90° )</td>
</tr>
<tr>
<td>4-2-3</td>
<td>[Diagram of equilateral triangle with angles A, B, and C]</td>
<td>( m\angle A = m\angle B = m\angle C = 60° )</td>
</tr>
</tbody>
</table>

You will prove Corollaries 4-2-2 and 4-2-3 in Exercises 24 and 25.
**Finding Angle Measures in Right Triangles**

One of the acute angles in a right triangle measures 22.9°. What is the measure of the other acute angle?

Let the acute angles be $\angle M$ and $\angle N$, with $m\angle M = 22.9^\circ$.

\[
m\angle M + m\angle N = 90^\circ \quad \text{Acute \ of \ rt. \ \Delta \ \text{are \ comp.}}
\]

\[
22.9 + m\angle N = 90 \quad \text{Substitute \ 22.9 \ \text{for} \ m\angle M.}
\]

\[
m\angle N = 67.1^\circ \quad \text{Subtract \ 22.9 \ \text{from both \ sides.}}
\]

The measure of one of the acute angles in a right triangle is given. What is the measure of the other acute angle?

2a. $63.7^\circ$  
2b. $x^\circ$  
2c. $48\frac{2}{5}^\circ$

The interior is the set of all points inside the figure. The exterior is the set of all points outside the figure. An interior angle is formed by two sides of a triangle. An exterior angle is formed by one side of the triangle and the extension of an adjacent side. Each exterior angle has two remote interior angles. A remote interior angle is an interior angle that is not adjacent to the exterior angle.

$\angle 4$ is an exterior angle. Its remote interior angles are $\angle 1$ and $\angle 2$.

**Theorem 4-2-4 Exterior Angle Theorem**

The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

\[m\angle 4 = m\angle 1 + m\angle 2\]

You will prove Theorem 4-2-4 in Exercise 28.

**Example 3**

**Applying the Exterior Angle Theorem**

Find $m\angle J$.

\[m\angle J + m\angle H = m\angle FGH \quad \text{Ext. \ \angle \ \text{Thm.}}
\]

\[5x + 17 + 6x - 1 = 126 \quad \text{Substitute} \ 5x + 17 \ \text{for} \ m\angle J, \ 6x - 1 \ \text{for} \ m\angle H, \ \text{and} \ 126 \ \text{for} \ m\angle FGH.
\]

\[11x + 16 = 126 \quad \text{Simplify.}
\]

\[11x = 110 \quad \text{Subtract \ 16 \ \text{from \ both \ sides.}}
\]

\[x = 10 \quad \text{Divide \ both \ sides \ by \ 11.}
\]

\[m\angle J = 5x + 17 = 5(10) + 17 = 67^\circ
\]

3. Find $m\angle ACD$. 

[Diagrams and equations related to the examples are present in the text.]
Theorem 4-2-5  Third Angles Theorem

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.</td>
<td>( \angle L \cong \angle R )</td>
<td>( \angle M \cong \angle N )</td>
</tr>
</tbody>
</table>

You will prove Theorem 4-2-5 in Exercise 27.

**Example 4**

Applying the Third Angles Theorem

Find \( m\angle C \) and \( m\angle F \).

\[
\begin{align*}
\angle C & \cong \angle F \\
m\angle C &= m\angle F \\
y^2 &= 3y^2 - 72 \\
-2y^2 &= -72 \\
y^2 &= 36 \\
So \ m\angle C &= 36^\circ. \\
Since \ m\angle F &= m\angle C, m\angle F &= 36^\circ.
\end{align*}
\]

**Check It Out!**

4. Find \( m\angle P \) and \( m\angle T \).

**Think and Discuss**

1. Use the Triangle Sum Theorem to explain why the supplement of one of the angles of a triangle equals in measure the sum of the other two angles of the triangle. Support your answer with a sketch.

2. Sketch a triangle and draw all of its exterior angles. How many exterior angles are there at each vertex of the triangle? How many total exterior angles does the triangle have?

3. **Get Organized** Copy and complete the graphic organizer.

In each box, write each theorem in words and then draw a diagram to represent it.

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Words</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle Sum Theorem</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exterior Angle Theorem</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Third Angles Theorem</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4-2 Angle Relationships in Triangles

Exercises

**GUIDED PRACTICE**

**Vocabulary** Apply the vocabulary from this lesson to answer each question.

1. To remember the meaning of remote interior angle, think of a television remote control. What is another way to remember the term remote?

2. An exterior angle is drawn at vertex E of \( \triangle DEF \). What are its remote interior angles?

3. What do you call segments, rays, or lines that are added to a given diagram?

**Astronomy** Use the following information for Exercises 4 and 5.

An asterism is a group of stars that is easier to recognize than a constellation. One popular asterism is the Summer Triangle, which is composed of the stars Deneb, Altair, and Vega.

4. What is the value of \( y \)?

5. What is the measure of each angle in the Summer Triangle?

The measure of one of the acute angles in a right triangle is given. What is the measure of the other acute angle?

6. 20.8°

7. \( y^\circ \)

8. 24\( \frac{2}{3} \)

Find each angle measure.

9. \( m\angle M \)

10. \( m\angle L \)

11. In \( \triangle ABC \), \( m\angle A = 65^\circ \), and the measure of an exterior angle at \( C \) is 117°. Find \( m\angle B \) and the \( m\angle BCA \).

12. \( m\angle C \) and \( m\angle F \)

13. \( m\angle S \) and \( m\angle U \)

14. For \( \triangle ABC \) and \( \triangle XYZ \), \( m\angle A = m\angle X \) and \( m\angle B = m\angle Y \). Find the measures of \( \angle C \) and \( \angle Z \) if \( m\angle C = 4x + 7 \) and \( m\angle Z = 3(x + 5) \).
PRACTICE AND PROBLEM SOLVING

15. **Navigation** A sailor on ship A measures the angle between ship B and the pier and finds that it is 39°. A sailor on ship B measures the angle between ship A and the pier and finds that it is 57°. What is the measure of the angle between ships A and B?

The measure of one of the acute angles in a right triangle is given. What is the measure of the other acute angle?

16. \[76 \frac{1}{4}°\]  
17. \[2x°\]  
18. \[56.8°\]

Find each angle measure.

19. \[m∠XYZ\]

20. \[m∠C\]

21. \[m∠N\] and \[m∠P\]

22. \[m∠Q\] and \[m∠S\]

23. **Multi-Step** The measures of the angles of a triangle are in the ratio 1:4:7. What are the measures of the angles? (*Hint: Let x, 4x, and 7x represent the angle measures.*)

24. Complete the proof of Corollary 4-2-2.

**Given:** \(\triangle DEF\) with right \(∠F\)

**Prove:** \(∠D \) and \(∠E\) are complementary.

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (\triangle DEF) with rt. (∠F)</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. b. ?</td>
<td>2. Def. of rt. (∠)</td>
</tr>
<tr>
<td>3. (m∠D + m∠E + m∠F = 180°)</td>
<td>3. c. ?</td>
</tr>
<tr>
<td>4. (m∠D + m∠E + 90° = 180°)</td>
<td>4. d. ?</td>
</tr>
<tr>
<td>6. (∠D ) and (∠E) are comp.</td>
<td>6. f. ?</td>
</tr>
</tbody>
</table>

25. Prove Corollary 4-2-3 using two different methods of proof.

**Given:** \(\triangle ABC\) is equiangular.

**Prove:** \(m∠A = m∠B = m∠C = 60°\)

26. **Multi-Step** The measure of one acute angle in a right triangle is \(1 \frac{1}{4}\) times the measure of the other acute angle. What is the measure of the larger acute angle?

27. Write a two-column proof of the Third Angles Theorem.
   **Given:** ΔABC with exterior angle ∠ACD
   **Prove:** m∠ACD = m∠A + m∠B
   (Hint: ∠BCA and ∠DCA form a linear pair.)

    Find each angle measure.
29. ∠UXW
30. ∠UWY
31. ∠WZX
32. ∠XYZ

33. **Critical Thinking** What is the measure of any exterior angle of an equiangular triangle? What is the sum of the exterior angle measures?

34. Find m∠SRQ, given that ∠P ≅ ∠U, ∠Q ≅ ∠T, and m∠RST = 37.5°.

35. **Multi-Step** In a right triangle, one acute angle measure is 4 times the other acute angle measure. What is the measure of the smaller angle?

36. **Aviation** To study the forces of lift and drag, the Wright brothers built a glider, attached two ropes to it, and flew it like a kite. They modeled the two wind forces as the legs of a right triangle.
   a. What part of a right triangle is formed by each rope?
   b. Use the Triangle Sum Theorem to write an equation relating the angle measures in the right triangle.
   c. Simplify the equation from part b. What is the relationship between x and y?
   d. Use the Exterior Angle Theorem to write an expression for z in terms of x.
   e. If x = 37°, use your results from parts c and d to find y and z.

37. **Estimation** Draw a triangle and two exterior angles at each vertex. Estimate the measure of each angle. How are the exterior angles at each vertex related? Explain.

38. **Given:** \( AB \perp BD, BD \perp DC, \angle A \cong \angle C \)
   **Prove:** \( AD \parallel CB \)

39. **Write About It** A triangle has angle measures of 115°, 40°, and 25°. Explain how to find the measures of the triangle's exterior angles. Support your answer with a sketch.

40. This problem will prepare you for the Multi-Step TAKS Prep on page 238.
   One of the steps in making an origami crane involves folding a square sheet of paper into the shape shown.
   a. ∠DCE is a right angle. \( FC \) bisects ∠DCE, and \( BC \) bisects ∠FCE. Find m∠FCB.
   b. Use the Triangle Sum Theorem to find m∠CBE.
41. What is the value of \( x \)?
   - A 19
   - B 52
   - C 57
   - D 71

42. Find the value of \( s \).
   - F 23
   - G 28
   - H 34
   - J 56

43. \( \angle A \) and \( \angle B \) are the remote interior angles of \( \angle BCD \) in \( \triangle ABC \). Which of these equations must be true?
   - A \( m \angle A - 180^\circ = m \angle B \)
   - B \( m \angle A = 90^\circ - m \angle B \)
   - C \( m \angle BCD = m \angle BCA - m \angle A \)
   - D \( m \angle B = m \angle BCD - m \angle A \)

44. Extended Response The measures of the angles in a triangle are in the ratio 2:3:4. Describe how to use algebra to find the measures of these angles. Then find the measure of each angle and classify the triangle.

**CHALLENGE AND EXTEND**

45. An exterior angle of a triangle measures 117°. Its remote interior angles measure \((2y^2 + 7)^\circ\) and \((61 - y^2)^\circ\). Find the value of \( y \).

46. Two parallel lines are intersected by a transversal. What type of triangle is formed by the intersection of the angle bisectors of two same-side interior angles? Explain. (Hint: Use geometry software or construct a diagram of the angle bisectors of two same-side interior angles.)

47. Critical Thinking Explain why an exterior angle of a triangle cannot be congruent to a remote interior angle.

48. Probability The measure of each angle in a triangle is a multiple of 30°. What is the probability that the triangle has at least two congruent angles?

49. In \( \triangle ABC \), \( m \angle B \) is 5° less than \( \frac{1}{2} \) times \( m \angle A \). \( m \angle C \) is 5° less than \( \frac{2}{3} \) times \( m \angle A \). What is \( m \angle A \) in degrees?

**SPIRAL REVIEW**

Make a table to show the value of each function when \( x \) is \(-2, 0, 1, \) and \( 4 \). (Previous course)

<table>
<thead>
<tr>
<th>Function</th>
<th>( x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50. ( f(x) = 3x - 4 )</td>
<td>( x )</td>
</tr>
<tr>
<td>51. ( f(x) = x^2 + 1 )</td>
<td>( x )</td>
</tr>
<tr>
<td>52. ( f(x) = (x - 3)^2 + 5 )</td>
<td>( x )</td>
</tr>
</tbody>
</table>

53. Find the length of \( \overline{NQ} \). Name the theorem or postulate that justifies your answer. (Lesson 1-2)

Classify each triangle by its side lengths. (Lesson 4-1)

<table>
<thead>
<tr>
<th>Triangle</th>
<th>54. ( \triangle ACD )</th>
<th>55. ( \triangle BCD )</th>
<th>56. ( \triangle ABD )</th>
</tr>
</thead>
</table>

57. What if…? If \( CA = 8 \), What is the effect on the classification of \( \triangle ACD \)?
**4-3 Congruent Triangles**

**Objectives**
- Use properties of congruent triangles.
- Prove triangles congruent by using the definition of congruence.

**Vocabulary**
- corresponding angles
- corresponding sides
- congruent polygons

**Who uses this?**
Machinists used triangles to construct a model of the International Space Station’s support structure.

Geometric figures are congruent if they are the same size and shape. **Corresponding angles** and **corresponding sides** are in the same position in polygons with an equal number of sides. Two polygons are **congruent polygons** if and only if their corresponding angles and sides are congruent. Thus triangles that are the same size and shape are congruent.

**Properties of Congruent Polygons**

To name a polygon, write the vertices in consecutive order. For example, you can name polygon PQRS as QRSP or SRQP, but not as PRQS. In a congruence statement, the order of the vertices indicates the corresponding parts.

**Example 1**

△RST and △XYZ represent the triangles of the space station’s support structure. If △RST ≅ △XYZ, identify all pairs of congruent corresponding parts.

Angles: \(\angle R \equiv \angle X, \angle S \equiv \angle Y, \angle T \equiv \angle Z\)

Sides: \(RS \equiv XY, ST \equiv YZ, RT \equiv XZ\)

1. If polygon LMNP ≅ polygon EFGH, identify all pairs of corresponding congruent parts.
**Example 2**

Using Corresponding Parts of Congruent Triangles

Given: \( \triangle EFH \cong \triangle GFH \)

**A** Find the value of \( x \).

1. \( \angle FHE \) and \( \angle FHG \) are rt. \( \angle s \).
2. \( \angle FHE \cong \angle FHG \)
3. \( m \angle FHE = m \angle FHG \)
4. \( (6x - 12)^\circ = 90^\circ \)
5. \( 6x = 102 \)
6. \( x = 17 \)

**B** Find \( m \angle GHF \).

1. \( m \angle EFH + m \angle FHE + m \angle E = 180^\circ \) \( \triangle \) Sum Thm.
2. \( m \angle EFH + 90 + 21.6 = 180 \)
3. \( m \angle EFH = 111.6 = 108 \)
4. \( m \angle GHF = 68.4^\circ \)
5. \( m \angle GHF = 68.4^\circ \) \( \triangle \) Sum Thm.

---

**Example 3**

Proving Triangles Congruent

Given: \( \angle P \) and \( \angle M \) are right angles.
1. \( R \) is the midpoint of \( PM \).
2. \( PQ \cong MN \), \( QR \cong NR \)

Prove: \( \triangle PQR \cong \triangle MNR \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle P ) and ( \angle M ) are rt. ( \triangle )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle P \cong \angle M )</td>
<td>2. Rt. ( \angle \cong ) Thm.</td>
</tr>
<tr>
<td>3. ( \angle PRO \cong \angle MRN )</td>
<td>3. Vert. ( \triangle ) Thm.</td>
</tr>
<tr>
<td>4. ( \angle Q \cong \angle N )</td>
<td>4. Third ( \angle ) Thm.</td>
</tr>
<tr>
<td>5. ( R ) is the mdpt. of ( PM )</td>
<td>5. Given</td>
</tr>
<tr>
<td>6. ( PR \cong MR )</td>
<td>6. Def. of mdpt.</td>
</tr>
<tr>
<td>7. ( PQ \cong MN ), ( QR \cong NR )</td>
<td>7. Given</td>
</tr>
<tr>
<td>8. ( \triangle PQR \cong \triangle MNR )</td>
<td>8. Def. of ( \cong \ \triangle )</td>
</tr>
</tbody>
</table>

---

3. **Given:** \( AD \) bisects \( BE \).
   \( BE \) bisects \( AD \).
   \( AB \cong DE \), \( \angle A \cong \angle D \)

**Prove:** \( \triangle ABC \cong \triangle DEC \)
“With overlapping triangles, it helps me to redraw the triangles separately. That way I can mark what I know about one triangle without getting confused by the other one.”

**Example 4** Engineering Application

The bars that give structural support to a roller coaster form triangles. Since the angle measures and the lengths of the corresponding sides are the same, the triangles are congruent.

Given: $\overline{JK} \perp \overline{KL}$, $\overline{ML} \perp \overline{KL}$, $\angle KLI \cong \angle LKM$, $\overline{JK} \cong \overline{ML}$, $\overline{JL} \cong \overline{MK}$

Prove: $\triangle JKL \cong \triangle MLK$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{JK} \perp \overline{KL}$, $\overline{ML} \perp \overline{KL}$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle KLI \cong \angle LKM$</td>
<td>2. Def. of $\perp$ lines</td>
</tr>
<tr>
<td>3. $\angle JKL \cong \angle MLK$</td>
<td>3. Rt. $\angle \cong$ Thm.</td>
</tr>
<tr>
<td>4. $\angle KLI \cong \angle LMK$</td>
<td>4. Given</td>
</tr>
<tr>
<td>5. $\triangle KLI \cong \triangle LMK$</td>
<td>5. Third $\triangle$ Thm.</td>
</tr>
<tr>
<td>6. $\overline{JK} \cong \overline{ML}$, $\overline{JL} \cong \overline{MK}$</td>
<td>6. Given</td>
</tr>
<tr>
<td>7. $\overline{KL} \cong \overline{LK}$</td>
<td>7. Reflex. Prop. of $\cong$</td>
</tr>
<tr>
<td>8. $\triangle JKL \cong \triangle MLK$</td>
<td>8. Def. of $\cong \triangle$</td>
</tr>
</tbody>
</table>

4. Use the diagram to prove the following.
Given: $\overline{MK}$ bisects $\overline{JL}$, $\overline{JL}$ bisects $\overline{MK}$, $\overline{JK} \cong \overline{ML}$, $\overline{JK} \parallel \overline{ML}$
Prove: $\triangle JKN \cong \triangle LMN$

**Think and Discuss**

1. A roof truss is a triangular structure that supports a roof. How can you be sure that two roof trusses are the same size and shape?

2. **Get Organized** Copy and complete the graphic organizer. In each box, name the congruent corresponding parts.
GUIDED PRACTICE

Vocabulary  Apply the vocabulary from this lesson to answer each question.

1. An everyday meaning of corresponding is “matching.” How can this help you find the corresponding parts of two triangles?

2. If \( \triangle ABC \cong \triangle RST \), what angle corresponds to \( \angle S \)?

Given: \( \triangle RST \cong \triangle LMN \). Identify the congruent corresponding parts.

3. \( RS \cong ? \)
4. \( LN \cong ? \)
5. \( \angle S \cong ? \)
6. \( TS \cong ? \)
7. \( \angle L \cong ? \)
8. \( \angle N \cong ? \)

Given: \( \triangle FGH \cong \triangle JKL \). Find each value.

9. \( KL \)
10. \( x \)

11. Given: \( E \) is the midpoint of \( AC \) and \( BD \).
\( AB \cong CD \), \( AB \parallel CD \)
Prove: \( \triangle ABE \cong \triangle CDE \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( AB \parallel CD )</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. ( \angle ABE \cong \angle CDE ), ( \angle BAE \cong \angle DCE )</td>
<td>2. b. ?</td>
</tr>
<tr>
<td>3. ( AB \cong CD )</td>
<td>3. c. ?</td>
</tr>
<tr>
<td>4. ( E ) is the mdpt. of ( AC ) and ( BD ).</td>
<td>4. d. ?</td>
</tr>
<tr>
<td>5. e. ?</td>
<td>5. Def. of mdpt.</td>
</tr>
<tr>
<td>6. ( \angle AEB \cong \angle CED )</td>
<td>6. f. ?</td>
</tr>
<tr>
<td>7. ( \triangle ABE \cong \triangle CDE )</td>
<td>7. g. ?</td>
</tr>
</tbody>
</table>

12. Engineering  The McDonald Observatory has four research telescopes and is a leading center for astronomical study. Prove that the triangles that make up the observatory dome are congruent.

Given: \( SU \cong ST \cong SR \), \( TU \cong TR \), \( \angle UST \cong \angle RST \), and \( \angle U \cong \angle R \)
Prove: \( \triangle RTS \cong \triangle UTS \)
Given: Polygon \( CDEF \cong \text{polygon } KLMN. \) Identify the congruent corresponding parts.

13. \( \overline{DE} \cong ? \)  
14. \( \overline{KN} \cong ? \)
15. \( \angle F \cong ? \)  
16. \( \angle L \cong ? \)

Given: \( \triangle ABD \cong \triangle CBD \). Find each value.

17. \( m\angle C \)  
18. \( y \)

19. Given: \( \overline{MP} \) bisects \( \angle NMR \). \( P \) is the midpoint of \( \overline{NR} \). \( \overline{MN} \cong \overline{MR}, \overline{\angle N} \cong \overline{\angle R} \)

Prove: \( \triangle MNP \cong \triangle MRP \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle N \cong \angle R )</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. ( \overline{MP} ) bisects ( \angle NMR ).</td>
<td>2. b. ?</td>
</tr>
<tr>
<td>3. c. ?</td>
<td>3. Def. of ( \angle ) bisector</td>
</tr>
<tr>
<td>4. d. ?</td>
<td>4. Third ( \triangle ) Thm.</td>
</tr>
<tr>
<td>5. e. ?</td>
<td>5. e. ?</td>
</tr>
<tr>
<td>7. ( \overline{MN} \cong \overline{MR} )</td>
<td>7. g. ?</td>
</tr>
<tr>
<td>8. ( \overline{MP} \cong \overline{MP} )</td>
<td>8. h. ?</td>
</tr>
<tr>
<td>9. ( \triangle MNP \cong \triangle MRP )</td>
<td>9. Def. of ( \cong \triangle )</td>
</tr>
</tbody>
</table>

20. **Hobbies** In a garden, triangular flower beds are separated by straight rows of grass as shown.

Given: \( \angle ADC \) and \( \angle BCD \) are right angles. \( AC \cong BD, AD \cong BC \)  
\( \angle DAC \cong \angle CBD \)

Prove: \( \triangle ADC \cong \triangle BCD \)

21. For two triangles, the following corresponding parts are given: \( GS \cong KP, GR \cong KH, SR \cong PH \), \( \angle S \cong \angle P, \angle G \cong \angle K \), and \( \angle R \cong \angle H \).

Write three different congruence statements.

22. The two polygons in the diagram are congruent. Complete the following congruence statement for the polygons. \( \text{polygon } R \ ? \cong \text{polygon } V \ ? \)

Write and solve an equation for each of the following.

23. \( \triangle ABC \cong \triangle DEF \). \( AB = 2x - 10 \), and \( DE = x + 20 \).

Find the value of \( x \) and \( AB \).

24. \( \triangle JKL \cong \triangle MNP \). \( m\angle L = (x^2 + 10)^\circ \), and \( m\angle P = (2x^2 + 1)^\circ \). What is \( m\angle L \)?

25. Polygon \( ABCD \cong \text{polygon } PQRS \). \( BC = 6x + 5 \), and \( QR = 5x + 7 \).

Find the value of \( x \) and \( BC \).
26. This problem will prepare you for the Multi-Step TAKS Prep on page 238. Many origami models begin with a square piece of paper, $JKLM$, that is folded along both diagonals to make the creases shown. $JL$ and $MK$ are perpendicular bisectors of each other, and $\angle NML \cong \angle NKL$.

a. Explain how you know that $\overline{KL}$ and $\overline{ML}$ are congruent.

b. Prove $\triangle NML \cong \triangle NKL$.

27. Draw a diagram and then write a proof.
Given: $BD \perp AC$. $D$ is the midpoint of $\overline{AC}$. $\overline{AB} \cong \overline{CB}$, and $BD$ bisects $\angle ABC$.
Prove: $\triangle ABD \cong \triangle CBD$

28. Critical Thinking Draw two triangles that are not congruent but have an area of 4 cm$^2$ each.

29. //ERROR ANALYSIS// Given $\triangle MPQ \cong \triangle EDF$.
Two solutions for finding $\angle E$ are shown. Which is incorrect? Explain the error.

30. Write About It Given the diagram of the triangles, is there enough information to prove that $\triangle HKL$ is congruent to $\triangle YWX$? Explain.

31. Which congruence statement correctly indicates that the two given triangles are congruent?

- $\triangle ABC \cong \triangle EFD$
- $\triangle ABC \cong \triangle DEF$
- $\triangle ABC \cong \triangle FDE$
- $\triangle ABC \cong \triangle FED$

32. $\triangle MNP \cong \triangle RST$. What are the values of $x$ and $y$?

- $x = 26, y = 21\frac{1}{3}$
- $x = 25, y = 20\frac{2}{3}$
- $x = 27, y = 20$
- $x = 30\frac{1}{3}, y = 16\frac{2}{3}$

33. $\triangle ABC \cong \triangle XYZ$. $m\angle A = 47.1^\circ$, and $m\angle C = 13.8^\circ$. Find $m\angle Y$.

- $13.8$
- $42.9$
- $119.1$

34. $\triangle MNR \cong \triangle SPQ$, $NL = 18$, $SP = 33$, $SR = 10$, $RQ = 24$, and $QP = 30$. What is the perimeter of $\triangle MNR$?

- $79$
- $85$
- $97$
35. **Multi-Step** Given that the perimeter of $TUVW$ is 149 units, find the value of $x$. Is $\triangle TUV \cong \triangle TWV$? Explain.

36. **Multi-Step** Polygon $ABCD \cong$ polygon $EFGH$. $\angle A$ is a right angle. $m\angle E = (y^2 - 10)^\circ$, and $m\angle H = (2y^2 - 132)^\circ$. Find $m\angle D$.

37. **Given:** $\overline{RS} \cong \overline{RT}$, $\angle S \cong \angle T$  
   **Prove:** $\triangle RST \cong \triangle RTS$

**SPIRAL REVIEW**

Two number cubes are rolled. Find the probability of each outcome. *(Previous course)*

38. Both numbers rolled are even.  
39. The sum of the numbers rolled is 5.

Classify each angle by its measure. *(Lesson 1-3)*

40. $m\angle DOC = 40^\circ$  
41. $m\angle BOA = 90^\circ$  
42. $m\angle COA = 140^\circ$

Find each angle measure. *(Lesson 4-2)*

43. $\angle Q$  
44. $\angle P$  
45. $\angle QRS$

**Career Path**

**Q:** What math classes did you take in high school?  
**A:** Algebra 1 and 2, Geometry, Precalculus

**Q:** What kind of degree or certification will you receive?  
**A:** I will receive an associate’s degree in applied science. Then I will take an exam to be certified as an EMT or paramedic.

**Q:** How do you use math in your hands-on training?  
**A:** I calculate dosages based on body weight and age. I also calculate drug doses in milligrams per kilogram per hour or set up an IV drip to deliver medications at the correct rate.

**Q:** What are your future career plans?  
**A:** When I am certified, I can work for a private ambulance service or with a fire department. I could also work in a hospital, transporting critically ill patients by ambulance or helicopter.
**Triangles and Congruence**

**Origami** Origami is the Japanese art of paper folding. The Japanese word *origami* literally means “fold paper.” This ancient art form relies on properties of geometry to produce fascinating and beautiful shapes.

Each of the figures shows a step in making an origami swan from a square piece of paper. The final figure shows the creases of an origami swan that has been unfolded.

1. Use the fact that $ABCD$ is a square to classify $\triangle ABD$ by its side lengths and by its angle measures.

2. $DB$ bisects $\angle ABC$ and $\angle ADC$. $DE$ bisects $\angle ADB$. Find the measures of the angles in $\triangle EDB$. Explain how you found the measures.

3. Given that $DB$ bisects $\angle ABC$ and $\angle EDF$, $BE \cong BF$, and $DE \cong DF$, prove that $\triangle EDB \cong \triangle FDB$. 

---

**Step 1**

Fold the paper in half diagonally and crease it. Turn it over.

**Step 2**

Fold corners $A$ and $C$ to the center line and crease. Turn it over.

**Step 3**

Fold in half along the center crease so that $DE$ and $DF$ are together.

**Step 4**

Fold the narrow point upward at a $90^\circ$ angle and crease. Push in the fold so that the neck is inside the body.

**Step 5**

Fold the tip downward and crease. Push in the fold so that the head is inside the neck.

**Step 6**

Fold up the flap to form the wing.
Quiz for Lessons 4-1 Through 4-3

4-1 Classifying Triangles
Classify each triangle by its angle measures.
1. \( \triangle ACD \)
2. \( \triangle ABD \)
3. \( \triangle ADE \)

Classify each triangle by its side lengths.
4. \( \triangle PQR \)
5. \( \triangle PRS \)
6. \( \triangle PQS \)

4-2 Angle Relationships in Triangles
Find each angle measure.
7. \( m \angle M \)
8. \( m \angle ABC \)

9. A carpenter built a triangular support structure for a roof. Two of the angles of the structure measure 37° and 55°. Find the measure of \( \angle RTP \), the angle formed by the roof of the house and the roof of the patio.

4-3 Congruent Triangles
Given: \( \triangle JKL \cong \triangle DEF \). Identify the congruent corresponding parts.
10. \( KL \cong ? \)
11. \( DF \cong ? \)
12. \( \angle K \cong ? \)
13. \( \angle F \cong ? \)

Given: \( \triangle PQR \cong \triangle STU \). Find each value.
14. \( PQ \)
15. \( y \)
16. Given: \( \overrightarrow{AB} \parallel \overrightarrow{CD}, \overrightarrow{AB} \equiv CD, \overrightarrow{AC} \equiv BD \), \( \overrightarrow{AC} \perp \overrightarrow{CD}, \overrightarrow{DB} \perp \overrightarrow{AB} \)
Prove: \( \triangle ACD \cong \triangle DBA \)
Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overrightarrow{AB} \parallel \overrightarrow{CD} )</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. ( \angle BAD \cong \angle CDA )</td>
<td>2. b. ?</td>
</tr>
<tr>
<td>3. ( \overrightarrow{AC} \perp \overrightarrow{CD}, \overrightarrow{DB} \perp \overrightarrow{AB} )</td>
<td>3. c. ?</td>
</tr>
<tr>
<td>4. ( \angle ACD ) and ( \angle DBA ) are rt. ( \triangle )</td>
<td>4. d. ?</td>
</tr>
<tr>
<td>5. e. ?</td>
<td>5. Rt. ( \angle \cong ) Thm.</td>
</tr>
<tr>
<td>7. ( \overrightarrow{AB} \equiv CD, \overrightarrow{AC} \equiv BD )</td>
<td>7. g. ?</td>
</tr>
<tr>
<td>8. h. ?</td>
<td>8. Reflex Prop. of ( \cong )</td>
</tr>
<tr>
<td>9. ( \triangle ACD \cong \triangle DBA )</td>
<td>9. i. ?</td>
</tr>
</tbody>
</table>
Explore SSS and SAS Triangle Congruence

In Lesson 4-3, you used the definition of congruent triangles to prove triangles congruent. To use the definition, you need to prove that all three pairs of corresponding sides and all three pairs of corresponding angles are congruent.

In this lab, you will discover some shortcuts for proving triangles congruent.

**Activity 1**

1. Measure and cut six pieces from the straws: two that are 2 inches long, two that are 4 inches long, and two that are 5 inches long.

2. Cut two pieces of string that are each about 20 inches long.

3. Thread one piece of each size of straw onto a piece of string. Tie the ends of the string together so that the pieces of straw form a triangle.

4. Using the remaining pieces, try to make another triangle with the same side lengths that is *not* congruent to the first triangle.

**Try This**

1. Repeat Activity 1 using side lengths of your choice. Are your results the same?

2. Do you think it is possible to make two triangles that have the same side lengths but that are not congruent? Why or why not?

3. How does your answer to Problem 2 provide a shortcut for proving triangles congruent?

4. Complete the following conjecture based on your results. Two triangles are congruent if ______?_______.
**Activity 2**

1. Measure and cut two pieces from the straws: one that is 4 inches long and one that is 5 inches long.

2. Use a protractor to help you bend a paper clip to form a 30° angle.

3. Place the pieces of straw on the sides of the 30° angle. The straws will form two sides of your triangle.

4. Without changing the angle formed by the paper clip, use a piece of straw to make a third side for your triangle, cutting it to fit as necessary. Use additional paper clips or string to hold the straws together in a triangle.

**Try This**

5. Repeat Activity 2 using side lengths and an angle measure of your choice. Are your results the same?

6. Suppose you know two side lengths of a triangle and the measure of the angle between these sides. Can the length of the third side be any measure? Explain.

7. How does your answer to Problem 6 provide a shortcut for proving triangles congruent?

8. Use the two given sides and the given angle from Activity 2 to form a triangle that is not congruent to the triangle you formed. (Hint: One of the given sides does not have to be adjacent to the given angle.)

9. Complete the following conjecture based on your results. Two triangles are congruent if _______?_______.
4-4 Triangle Congruence: SSS and SAS

Objectives
Apply SSS and SAS to construct triangles and to solve problems.
Prove triangles congruent by using SSS and SAS.

Vocabulary
triangle rigidity
included angle

Who uses this?
Engineers used the property of triangle rigidity to design the internal support for the Statue of Liberty and to build bridges, towers, and other structures. (See Example 2.)

In Lesson 4-3, you proved triangles congruent by showing that all six pairs of corresponding parts were congruent.

The property of triangle rigidity gives you a shortcut for proving two triangles congruent. It states that if the side lengths of a triangle are given, the triangle can have only one shape.

For example, you only need to know that two triangles have three pairs of congruent corresponding sides. This can be expressed as the following postulate.

Postulate 4-4-1 Side-Side-Side (SSS) Congruence

POSTULATE | HYPOTHESIS | CONCLUSION
--- | --- | ---
If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

△ABC ≅ △FDE

EXAMPLE 1

Remember!
Adjacent triangles share a side, so you can apply the Reflexive Property to get a pair of congruent parts.

Using SSS to Prove Triangle Congruence

Use SSS to explain why △PQR ≅ △PSR.
It is given that PQ ≅ PS and that QR ≅ SR. By the Reflexive Property of Congruence, PR ≅ PR. Therefore △PQR ≅ △PSR by SSS.

An included angle is an angle formed by two adjacent sides of a polygon. ∠B is the included angle between sides AB and BC.
It can also be shown that only two pairs of congruent corresponding sides are needed to prove the congruence of two triangles if the included angles are also congruent.

**Postulate 4-4-2 Side-Angle-Side (SAS) Congruence**

<table>
<thead>
<tr>
<th>POSTULATE</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.</td>
<td>△ABC ≅ △EFD</td>
<td></td>
</tr>
</tbody>
</table>

**EXAMPLE 2 Engineering Application**

The figure shows part of the support structure of the Statue of Liberty. Use SAS to explain why △KPN ≅ △LPM.

It is given that KP ≅ LP and that NP ≅ MP. By the Vertical Angles Theorem, ∠KPN ≅ ∠LPM. Therefore △KPN ≅ △LPM by SAS.

2. Use SAS to explain why △ABC ≅ △DBC.

The SAS Postulate guarantees that if you are given the lengths of two sides and the measure of the included angle, you can construct one and only one triangle.

**Construction Congruent Triangles Using SAS**

Use a straightedge to draw two segments and one angle, or copy the given segments and angle.

1. Construct AB congruent to one of the segments.
2. Construct ∠A congruent to the given angle.
3. Construct AC congruent to the other segment. Draw CB to complete △ABC.
EXAMPLE 3

Verifying Triangle Congruence

Show that the triangles are congruent for the given value of the variable.

A \( \triangle UVW \cong \triangle YXW, x = 3 \)

\[
\begin{align*}
ZY &= x - 1 \\
&= 3 - 1 = 2 \\
XZ &= x = 3 \\
XY &= 3x - 5 \\
&= 3(3) - 5 = 4 \\
\end{align*}
\]

\( \overline{UV} \cong \overline{YX}, \overline{VW} \cong \overline{XZ}, \) and \( \overline{UW} \cong \overline{YZ} \).

So \( \triangle UVW \cong \triangle YXZ \) by SSS.

B \( \triangle DEF \cong \triangle JGH, y = 7 \)

\[
\begin{align*}
JG &= 2y + 1 \\
&= 2(7) + 1 \\
&= 15 \\
GH &= y^2 - 4y + 3 \\
&= (7)^2 - 4(7) + 3 \\
&= 24 \\
m\angle G &= 12y + 42 \\
&= 12(7) + 42 \\
&= 126° \\
\end{align*}
\]

\( \overline{DE} \cong \overline{JG}, \overline{EF} \cong \overline{GH}, \) and \( \angle E \cong \angle G \).

So \( \triangle DEF \cong \triangle JGH \) by SAS.

3. Show that \( \triangle ADB \cong \triangle CDB \) when \( t = 4 \).

EXAMPLE 4

Proving Triangles Congruent

Given: \( \ell \parallel m, \overline{EG} \cong \overline{HF} \)

Prove: \( \triangle EGF \cong \triangle HFG \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{EG} \cong \overline{HF} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \ell \parallel m )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \angle EGF \cong \angle HFG )</td>
<td>3. Alt. Int. ( \triangle ) Thm.</td>
</tr>
<tr>
<td>4. ( \overline{FG} \cong \overline{GF} )</td>
<td>4. Reflex Prop. of ( \cong )</td>
</tr>
<tr>
<td>5. ( \triangle EGF \cong \triangle HFG )</td>
<td>5. SAS Steps 1, 3, 4</td>
</tr>
</tbody>
</table>

4. Given: \( \overline{QP} \) bisects \( \angle RQS \). \( \overline{QR} \cong \overline{QS} \)

Prove: \( \triangle RQP \cong \triangle SQP \)
GUIDED PRACTICE

1. **Vocabulary** In $\triangle RST$ which angle is the included angle of sides $\overline{ST}$ and $\overline{TR}$?

2. Use SSS to explain why the triangles in each pair are congruent.
   - $\triangle ABD \cong \triangle CDB$
   - $\triangle MNP \cong \triangle MQP$

3. **Design** This Texas flag consists of a blue, perpendicular stripe with a white star in the center. The star consists of five triangles. $GJ = LG = 20$ in., and $GK = GH = 13$ in. Use SAS to explain why $\triangle JGK \cong \triangle LGH$.

Show that the triangles are congruent for the given value of the variable.

4. $\triangle GHJ \cong \triangle IHJ$, $x = 4$

5. $\triangle RST \cong \triangle TUR$, $x = 18$
7. Given: $\overline{JK} \cong \overline{ML}$, $\angle JKL \cong \angle MLK$
Prove: $\triangle JKL \cong \triangle MLK$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{JK} \cong \overline{ML}$</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. b. ?</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. $KL \cong LK$</td>
<td>3. c. ?</td>
</tr>
<tr>
<td>4. $\triangle JKL \cong \triangle MLK$</td>
<td>4. d. ?</td>
</tr>
</tbody>
</table>

PRACTICE AND PROBLEM SOLVING

Use SSS to explain why the triangles in each pair are congruent.

8. $\triangle BCD \cong \triangle EDC$

9. $\triangle GJK \cong \triangle GJL$

10. **Theater** The lights shining on a stage appear to form two congruent right triangles. Given $\overline{EC} \cong \overline{DB}$, use SAS to explain why $\triangle ECB \cong \triangle DBC$.

Show that the triangles are congruent for the given value of the variable.

11. $\triangle MNP \cong \triangle QNP$, $y = 3$

12. $\triangle XYZ \cong \triangle STU$, $t = 5$

13. Given: $B$ is the midpoint of $\overline{DC}$, $\overline{AB} \perp \overline{DC}$
Prove: $\triangle ABD \cong \triangle ABC$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $B$ is the mdpt. of $\overline{DC}$</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. b. ?</td>
<td>2. Def. of mdpt.</td>
</tr>
<tr>
<td>3. c. ?</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. $\angle ABD$ and $\angle ABC$ are rt. $\triangle$</td>
<td>4. d. ?</td>
</tr>
<tr>
<td>5. $\angle ABD \cong \angle ABC$</td>
<td>5. e. ?</td>
</tr>
<tr>
<td>7. $\triangle ABD \cong \triangle ABC$</td>
<td>7. g. ?</td>
</tr>
</tbody>
</table>
Which postulate, if any, can be used to prove the triangles congruent?

14. 

16. 

17. 

18. Explain what additional information, if any, you would need to prove \( \triangle ABC \cong \triangle DEC \) by each postulate.
   a. SSS
   b. SAS

**Multi-Step** Graph each triangle. Then use the Distance Formula and the SSS Postulate to determine whether the triangles are congruent.

19. \( \triangle QRS \) and \( \triangle TUV \)
   
   \( Q(-2, 0), R(1, -2), S(-3, -2) \)
   
   \( T(5, 1), U(3, -2), V(3, 2) \)

20. \( \triangle ABC \) and \( \triangle DEF \)
   
   \( A(2, 3), B(3, -1), C(7, 2) \)
   
   \( D(-3, 1), E(1, 2), F(-3, 5) \)

21. Given: \( \angle ZVY \cong \angle WYV, \) 
   \( \angle ZWV \cong \angle WYZ, \) 
   \( \overline{VW} \cong \overline{YZ} \)

Prove: \( \triangle ZVY \cong \triangle WYV \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle ZVY \cong \angle WYV, \angle ZWV \cong \angle WYZ )</td>
<td>a. ?</td>
</tr>
<tr>
<td>2. ( \angle ZVY = \angle WYV, \angle ZWV = \angle WYZ )</td>
<td>b. ?</td>
</tr>
<tr>
<td>3. ( \angle ZVY + \angle ZWV = \angle WYV + \angle WYZ )</td>
<td>c. ?</td>
</tr>
<tr>
<td>4. ( \angle ZVY \cong \angle WYV )</td>
<td>d. ?</td>
</tr>
<tr>
<td>5. ( \angle ZWV \cong \angle WYZ )</td>
<td>e. ?</td>
</tr>
<tr>
<td>6. ( \overline{VW} \cong \overline{YZ} )</td>
<td>f. ?</td>
</tr>
<tr>
<td>7. ( \triangle ZVY \cong \triangle WYV )</td>
<td>g. ?</td>
</tr>
</tbody>
</table>

22. This problem will prepare you for the Multi-Step TAKS Prep on page 280. The diagram shows two triangular trusses that were built for the roof of a doghouse.

   a. You can use a protractor to check that \( \angle A \) and \( \angle D \) are right angles. Explain how you could make just two additional measurements on each truss to ensure that the trusses are congruent.

   b. You verify that the trusses are congruent and find that \( AB = AC = 2.5 \text{ ft} \). Find the length of \( EF \) to the nearest tenth. Explain.
23. **Critical Thinking** Draw two isosceles triangles that are not congruent but that have a perimeter of 15 cm each.

24. \( \triangle ABC \cong \triangle ADC \) for what value of \( x \)? Explain why the SSS Postulate can be used to prove the two triangles congruent.

25. **Ecology** A wing deflector is a triangular structure made of logs that is filled with large rocks and placed in a stream to guide the current or prevent erosion. Wing deflectors are often used in pairs. Suppose an engineer wants to build two wing deflectors. The logs that form the sides of each wing deflector are perpendicular. How can the engineer make sure that the two wing deflectors are congruent?

26. **Write About It** If you use the same two sides and included angle to repeat the construction of a triangle, are your two constructed triangles congruent? Explain.

27. **Construction** Use three segments (SSS) to construct a scalene triangle. Suppose you then use the same segments in a different order to construct a second triangle. Will the result be the same? Explain.

28. Which of the three triangles below can be proven congruent by SSS or SAS?

   ![Triangle Diagrams]

   - **A** I and II
   - **B** II and III
   - **C** I and III
   - **D** I, II, and III

29. What is the perimeter of polygon \( ABCD \)?

   - **F** 29.9 cm
   - **G** 39.8 cm
   - **H** 49.8 cm
   - **J** 59.8 cm

30. Jacob wants to prove that \( \triangle FGH \cong \triangle JKL \) using SAS. He knows that \( \overline{FG} \cong \overline{JK} \) and \( \overline{FH} \cong \overline{JL} \). What additional piece of information does he need?

   - **A** \( \angle F \cong \angle J \)
   - **B** \( \angle G \cong \angle K \)
   - **C** \( \angle H \cong \angle L \)
   - **D** \( \angle F \cong \angle G \)

31. What must the value of \( x \) be in order to prove that \( \triangle EFG \cong \triangle EHG \) by SSS?

   - **F** 1.5
   - **G** 4.25
   - **H** 4.67
   - **I** 5.5

---

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**Challenge and Extend**

32. Given: \( \angle ADC \) and \( \angle BCD \) are supplementary. \( AD \cong CB \)
   Prove: \( \triangle ADB \cong \triangle CBD \)
   (Hint: Draw an auxiliary line.)

33. Given: \( \angle QPS \cong \angle TPR \), \( PQ \cong PT \), \( PR \cong PS \)
   Prove: \( \triangle PQR \cong \triangle PTS \)

**Algebra** Use the following information for Exercises 34 and 35.
Find the value of \( x \). Then use SSS or SAS to write a paragraph proof showing that two of the triangles are congruent.

34. \( m\angle FJK = 2x^\circ \)
   \( m\angle KFJ = (3x + 10)^\circ \)
   \( KJ = 4x + 8 \)
   \( HJ = 6(x - 4) \)

35. \( \overline{FJ} \) bisects \( \angle KFH \).
   \( m\angle FJ = (2x + 6)^\circ \)
   \( m\angle HFJ = (3x - 21)^\circ \)
   \( FK = 8x - 45 \)
   \( FH = 6x + 9 \)

**Spiral Review**

Solve and graph each inequality. *(Previous course)*

36. \( \frac{x}{2} - 8 \leq 5 \)
37. \( 2a + 4 > 3a \)
38. \(-6m - 1 \leq -13 \)

Solve each equation. Write a justification for each step. *(Lesson 2-5)*

39. \( 4x - 7 = 21 \)
40. \( \frac{a}{4} + 5 = -8 \)
41. \( 6r = 4r + 10 \)

Given: \( \triangle EFG \cong \triangle GHE \). Find each value. *(Lesson 4-3)*

42. \( x \)
43. \( m\angle FEG \)
44. \( m\angle FGH \)

**Using Technology**

Use geometry software to complete the following.

1. Draw a triangle and label the vertices \( A, B, \) and \( C \).
   Draw a point and label it \( D \). Mark a vector from \( A \) to \( B \) and translate \( D \) by the marked vector. Label the image \( E \).
   Draw \( \overline{DE} \). Mark \( \angle BAC \) and rotate \( \overline{DE} \) about \( D \) by the marked angle. Mark \( \angle ABC \) and rotate \( \overline{DE} \) about \( E \) by the marked angle. Label the intersection \( F \).

2. Drag \( A, B, \) and \( C \) to different locations.
   What do you notice about the two triangles?

3. Write a conjecture about \( \triangle ABC \) and \( \triangle DEF \).

4. Test your conjecture by measuring the sides and angles of \( \triangle ABC \) and \( \triangle DEF \).
Predict Other Triangle Congruence Relationships

Geometry software can help you investigate whether certain combinations of triangle parts will make only one triangle. If a combination makes only one triangle, then this arrangement can be used to prove two triangles congruent.

Activity 1

1. Construct $\angle CAB$ measuring $45^\circ$ and $\angle EDF$ measuring $110^\circ$.

2. Move $\angle EDF$ so that $\overline{DE}$ overlays $\overline{BA}$. Where $\overline{DF}$ and $\overline{AC}$ intersect, label the point $G$. Measure $\angle DGA$.

3. Move $\angle CAB$ to the left and right without changing the measures of the angles. Observe what happens to the size of $\angle DGA$.

4. Measure the distance from $A$ to $D$. Try to change the shape of the triangle without changing $\overline{AD}$ and the measures of $\angle A$ and $\angle D$.

Try This

1. Repeat Activity 1 using angle measures of your choice. Are your results the same? Explain.
2. Do the results change if one of the given angles measures $90^\circ$?
3. What theorem proves that the measure of $\angle DGA$ in Step 2 will always be the same?
4. In Step 3 of the activity, the angle measures in $\triangle ADG$ stayed the same as the size of the triangle changed. Does Angle-Angle-Angle, like Side-Side-Side, make only one triangle? Explain.
5. Repeat Step 4 of the activity but measure the length of $\overline{AG}$ instead of $\overline{AD}$. Are your results the same? Does this lead to a new congruence postulate or theorem?
6. If you are given two angles of a triangle, what additional piece of information is needed so that only one triangle is made? Make a conjecture based on your findings in Step 5.
**Activity 2**

1. Construct $\overline{YZ}$ with a length of 6.5 cm.

2. Using $\overline{YZ}$ as a side, construct $\angle XYZ$ measuring 43°.

3. Draw a circle at $Z$ with a radius of 5 cm. Construct $\overline{ZW}$, a radius of circle $Z$.

4. Move $W$ around circle $Z$. Observe the possible shapes of $\triangle YZW$.

**Try This**

7. In Step 4 of the activity, how many different triangles were possible? Does Side-Side-Angle make only one triangle?

8. Repeat Activity 2 using an angle measure of 90° in Step 2 and a circle with a radius of 7 cm in Step 3. How many different triangles are possible in Step 4?

9. Repeat the activity again using a measure of 90° in Step 2 and a circle with a radius of 8.25 cm in Step 3. Classify the resulting triangle by its angle measures.

10. Based on your results, complete the following conjecture. In a Side-Side-Angle combination, if the corresponding nonincluded angles are ____?____, then only one triangle is possible.
4-5 Triangle Congruence: ASA, AAS, and HL

Objectives
Apply ASA, AAS, and HL to construct triangles and to solve problems.
Prove triangles congruent by using ASA, AAS, and HL.

Vocabulary
included side

Why use this?
Bearings are used to convey direction, helping people find their way to specific locations.

Participants in an orienteering race use a map and a compass to find their way to checkpoints along an unfamiliar course. Directions are given by bearings, which are based on compass headings. For example, to travel along the bearing S 43° E, you face south and then turn 43° to the east.

An included side is the common side of two consecutive angles in a polygon. The following postulate uses the idea of an included side.

Postulate 4-5-1
Angle-Side-Angle (ASA) Congruence

<table>
<thead>
<tr>
<th>POSTULATE</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Problem-Solving Application

Organizers of an orienteering race are planning a course with checkpoints A, B, and C. Does the table give enough information to determine the location of the checkpoints?

1 Understand the Problem

The answer is whether the information in the table can be used to find the position of checkpoints A, B, and C. List the important information: The bearing from A to B is N 55° E. From B to C is N 26° W, and from C to A is S 20° W. The distance from A to B is 7.6 km.
**2 Make a Plan**

Draw the course using vertical lines to show north-south directions. Then use these parallel lines and the alternate interior angles to help find angle measures of $\triangle ABC$.

**3 Solve**

$m\angle CAB = 55^\circ - 20^\circ = 35^\circ$

$m\angle CBA = 180^\circ - (26^\circ + 55^\circ) = 99^\circ$

You know the measures of $\angle CAB$ and $\angle CBA$ and the length of the included side $AB$. Therefore by ASA, a unique triangle $ABC$ is determined.

**4 Look Back**

One and only one triangle can be made using the information in the table, so the table does give enough information to determine the location of all the checkpoints.

1. **What if...?** If 7.6 km is the distance from $B$ to $C$, is there enough information to determine the location of all the checkpoints? Explain.

---

**EXAMPLE 2 Applying ASA Congruence**

Determine if you can use ASA to prove $\triangle UVX \cong \triangle WVX$. Explain.

$\angle UXV \cong \angle WXV$ as given. Since $\angle WVX$ is a right angle that forms a linear pair with $\angle UVX$, $\angle UWV \cong \angle UVX$. Also $\overline{VX} \cong \overline{VX}$ by the Reflexive Property. Therefore $\triangle UVX \cong \triangle WVX$ by ASA.

2. Determine if you can use ASA to prove $\triangle NKL \cong \triangle LMN$. Explain.

---

**Construction Congruent Triangles Using ASA**

Use a straightedge to draw a segment and two angles, or copy the given segment and angles.

1. Construct $\overline{CD}$ congruent to the given segment.

2. Construct $\angle C$ congruent to one of the angles.

3. Construct $\angle D$ congruent to the other angle.

4. Label the intersection of the rays as $E$.

---

4-5 Triangle Congruence: ASA, AAS, and HL
You can use the Third Angles Theorem to prove another congruence relationship based on ASA. This theorem is Angle-Angle-Side (AAS).

**Theorem 4-5-2 Angle-Angle-Side (AAS) Congruence**

**THEOREM**

If two angles and a nonincluded side of one triangle are congruent to the corresponding angles and nonincluded side of another triangle, then the triangles are congruent.

**HYPOTHESIS**

\[ \triangle GHJ \cong \triangle KLM \]

**CONCLUSION**

\[ \triangle GHJ \cong \triangle KLM \]

**Proof**

Given: \( \angle G \cong \angle K, \angle J \cong \angle M, \overline{HJ} \cong \overline{LM} \)

Prove: \( \triangle GHJ \cong \triangle KLM \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle G \cong \angle K, \angle J \cong \angle M )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle H \cong \angle L )</td>
<td>2. Third ( \triangle ) Thm.</td>
</tr>
<tr>
<td>3. ( \overline{HJ} \cong \overline{LM} )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \triangle GHJ \cong \triangle KLM )</td>
<td>4. ASA Steps 1, 3, and 2</td>
</tr>
</tbody>
</table>

**Example 3 Using AAS to Prove Triangles Congruent**

Use AAS to prove the triangles congruent.

Given: \( \overline{AB} \parallel \overline{ED}, \overline{BC} \cong \overline{DC} \)

Prove: \( \triangle ABC \cong \triangle EDC \)

Proof:

\[ \overline{BC} \cong \overline{DC} \]

Given

\[ \angle B \cong \angle D \]

Alt. Int. \( \triangle \) Thm.

\[ \triangle ABC \cong \triangle EDC \]

AAS

\[ \overline{AB} \parallel \overline{ED} \]

Given

\[ \angle A \cong \angle E \]

Alt. Int. \( \triangle \) Thm.

There are four theorems for right triangles that are not used for acute or obtuse triangles. They are Leg-Leg (LL), Hypotenuse-Angle (HA), Leg-Angle (LA), and Hypotenuse-Leg (HL). You will prove LL, HA, and LA in Exercises 21, 23, and 33.
Theorem 4-5-3  
**Hypotenuse-Leg (HL) Congruence**

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.</td>
<td><img src="image" alt="Diagram" /></td>
<td>△ABC ≅ △DEF</td>
</tr>
</tbody>
</table>

You will prove the Hypotenuse-Leg Theorem in Lesson 4-8.

**Example 4**  
**Applying HL Congruence**  
Determine if you can use the HL Congruence Theorem to prove the triangles congruent. If not, tell what else you need to know.

A  
△VWX and △YXW  
According to the diagram, △VWX and △YXW are right triangles that share hypotenuse WX. WX ≅ XW by the Reflexive Property. It is given that WV ≅ XY, therefore △VWX ≅ △YXW by HL.

B  
△VWZ and △YXZ  
This conclusion cannot be proved by HL. According to the diagram, △VWZ and △YXZ are right triangles, and WV ≅ XY. You do not know that hypotenuse WZ is congruent to hypotenuse XZ.

4. Determine if you can use the HL Congruence Theorem to prove △ABC ≅ △DCB. If not, tell what else you need to know.

**Think and Discuss**

1. Could you use AAS to prove that these two triangles are congruent? Explain.
![Diagram](image)

2. The arrangement of the letters in ASA matches the arrangement of what parts of congruent triangles? Include a sketch to support your answer.

3. **Get Organized**  
Copy and complete the graphic organizer.  
In each column, write a description of the method and then sketch two triangles, marking the appropriate congruent parts.

<table>
<thead>
<tr>
<th>Proving Triangles Congruent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Def. of △ ≅</td>
</tr>
<tr>
<td>Words</td>
</tr>
<tr>
<td>Pictures</td>
</tr>
</tbody>
</table>
GUIDED PRACTICE

1. **Vocabulary** A triangle contains \( \angle ABC \) and \( \angle ACB \) with \( BC \) “closed in” between them. How would this help you remember the definition of *included side*?

**Surveying** Use the table for Exercises 2 and 3.

A landscape designer surveyed the boundaries of a triangular park. She made the following table for the dimensions of the land.

<table>
<thead>
<tr>
<th>Bearing</th>
<th>A to B</th>
<th>B to C</th>
<th>C to A</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>S 25° E</td>
<td>N 62° W</td>
<td>115 ft</td>
</tr>
</tbody>
</table>

2. Draw the plot of land described by the table. Label the measures of the angles in the triangle.

3. Does the table have enough information to determine the locations of points \( A, B, \) and \( C \)? Explain.

Determine if you can use ASA to prove the triangles congruent. Explain.

4. \( \triangle VRS \) and \( \triangle VTS \), given that \( VS \) bisects \( \angle RST \) and \( \angle RVT \)

5. \( \triangle DEH \) and \( \triangle FGH \)

6. Use AAS to prove the triangles congruent.

   Given: \( \angle R \) and \( \angle P \) are right angles.
   \( QR \parallel SP \)
   \( \angle R \) and \( \angle P \) are rt. \( \Delta \)
   \( \angle R \equiv \angle P \)

   Prove: \( \triangle QPS \equiv \triangle SRQ \)

   Proof:

7. \( \triangle ABC \) and \( \triangle CDA \)

8. \( \triangle XYV \) and \( \triangle ZYV \)
PRACTICE AND PROBLEM SOLVING

Surveying Use the table for Exercises 9 and 10.
From two different observation towers a fire is sighted. The locations of the towers are given in the following table.

<table>
<thead>
<tr>
<th>X to Y</th>
<th>X to F</th>
<th>Y to F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing</td>
<td>E</td>
<td>N 53° E</td>
</tr>
<tr>
<td>Distance</td>
<td>6 km</td>
<td>?</td>
</tr>
</tbody>
</table>

9. Draw the diagram formed by observation tower X, observation tower Y, and the fire F. Label the measures of the angles.

10. Is there enough information given in the table to pinpoint the location of the fire? Explain.

Determine if you can use ASA to prove the triangles congruent. Explain.

11. △MKJ and △MKL

12. △RST and △TUR

13. Given: \(AB \cong DE\), \(∠C \cong ∠F\)
   Prove: △ABC \(\cong\) △DEF
   
   Proof:
   
   \[\angle A \text{ and } \angle D \text{ are rt. }\]
   a. ?
   Given
   
   \[\text{Rt. } ∠ \text{ Thm.}\]
   b. ?
   
   \[AB \cong DE\]
   c. ?
   Given
   
   \[△ABC \cong △DEF\]
   d. ?

Determine if you can use the HL Congruence Theorem to prove the triangles congruent. If not, tell what else you need to know.

14. △GHJ and △JKG

15. △ABE and △DCE, given that E is the midpoint of AD and BC

Multi-Step For each pair of triangles write a triangle congruence statement. Identify the transformation that moves one triangle to the position of the other triangle.

16.

17.

18. Critical Thinking Side-Side-Angle (SSA) cannot be used to prove two triangles congruent. Draw a diagram that shows why this is true.
19. This problem will prepare you for the Multi-Step TAKS Prep on page 280. A carpenter built a truss to support the roof of a doghouse.
   a. The carpenter knows that \( KJ \cong MJ \). Can the carpenter conclude that \( \triangle KJL \cong \triangle MJL \)? Why or why not?
   b. Suppose the carpenter also knows that \( \angle JLK \) is a right angle. Which theorem can be used to show that \( \triangle KJL \cong \triangle MJL \)?

20. /// ERROR ANALYSIS /// Two proofs that \( \triangle EFH \cong \triangle GHF \) are given. Which is incorrect? Explain the error.

21. Write a paragraph proof of the Leg-Leg (LL) Congruence Theorem. If the legs of one right triangle are congruent to the corresponding legs of another right triangle, the triangles are congruent.

22. Use AAS to prove the triangles congruent.
   Given: \( AD \parallel BC, AD \cong CB \)
   Prove: \( \triangle AED \cong \triangle CEB \)
   Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( AD \parallel BC )</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. ( \angle DAE \cong \angle BCE )</td>
<td>2. b. ?</td>
</tr>
<tr>
<td>3. c. ?</td>
<td>3. Vert. ( \triangle ) Thm.</td>
</tr>
<tr>
<td>5. e. ?</td>
<td>4. f. ?</td>
</tr>
</tbody>
</table>

23. Prove the Hypotenuse-Angle (HA) Theorem.
   Given: \( KM \perp JL, JM \cong LM, \angle JMK \cong \angle LMK \)
   Prove: \( \triangle JKM \cong \triangle LKM \)

24. Write About It The legs of both right \( \triangle DEF \) and right \( \triangle RST \) are 3 cm and 4 cm. They each have a hypotenuse 5 cm in length. Describe two different ways you could prove that \( \triangle DEF \cong \triangle RST \).

25. Construction Use the method for constructing perpendicular lines to construct a right triangle.

26. What additional congruence statement is necessary to prove \( \triangle XWY \cong \triangle XVZ \) by ASA?
   \[ \begin{align*}
   & A \quad \angle XVZ \cong \angle XWY \\
   & B \quad \angle VUY \cong \angle WUZ
   \end{align*} \]
27. Which postulate or theorem justifies the congruence statement \( \triangle STU \cong \triangle VUT \)?
   - (F) ASA  
   - (H) HL  
   - (G) SSS  
   - (J) SAS

28. Which of the following congruence statements is true?
   - (A) \( \angle A \cong \angle B \)  
   - (C) \( \triangle AED \cong \triangle CEB \)  
   - (B) \( CE \cong DE \)  
   - (D) \( \triangle AED \cong \triangle BEC \)

29. In \( \triangle RST \), \( RT = 6y - 2 \). In \( \triangle UVW \), \( UW = 2y + 7 \). \( \angle R \cong \angle U \), and \( \angle S \cong \angle V \).
   What must be the value of \( y \) in order to prove that \( \triangle RST \cong \triangle UVW \)?
   - (F) 1.25  
   - (G) 2.25  
   - (H) 9.0  
   - (J) 11.5

30. **Extended Response** Draw a triangle. Construct a second triangle that has the same angle measures but is not congruent. Compare the lengths of each pair of corresponding sides. Consider the relationship between the lengths of the sides and the measures of the angles. Explain why Angle-Angle-Angle (AAA) is not a congruence principle.

### CHALLENGE AND EXTEND

31. **Sports** This bicycle frame includes \( \triangle VSU \) and \( \triangle VTU \), which lie in intersecting planes. From the given angle measures, can you conclude that \( \triangle VSU \cong \triangle VTU \)? Explain.
   \[ m\angle VUS = (7y - 2)° \quad m\angle VUT = \left(\frac{5}{2}x - \frac{1}{2}\right)° \]
   \[ m\angle USV = \left(\frac{5}{3}y\right)° \quad m\angle UTV = (4x + 8)° \]
   \[ m\angle SVU = (3y - 6)° \quad m\angle TVU = 2x° \]

32. **Given:** \( \triangle ABC \) is equilateral. \( C \) is the midpoint of \( DE \). \( \angle DAC \) and \( \angle EBC \) are congruent and supplementary.
   **Prove:** \( \triangle DAC \cong \triangle EBC \)

33. Write a two-column proof of the Leg-Angle (LA) Congruence Theorem. If a leg and an acute angle of one right triangle are congruent to the corresponding parts of another right triangle, the triangles are congruent. *(Hint: There are two cases to consider.)*

34. If two triangles are congruent by ASA, what theorem could you use to prove that the triangles are also congruent by AAS? Explain.

### SPIRAL REVIEW

Identify the \( x \)- and \( y \)-intercepts. Use them to graph each line. *(Previous course)*

35. \( y = 3x - 6 \)  
36. \( y = \frac{1}{2}x + 4 \)  
37. \( y = -5x + 5 \)

38. Find \( AB \) and \( BC \) if \( AC = 10 \). *(Lesson 1-6)*

39. Find \( m\angle C \). *(Lesson 4-2)*
Chapter 4 Triangle Congruence

### Objective
Use CPCTC to prove parts of triangles are congruent.

### Vocabulary
CPCTC

#### Why learn this?
You can use congruent triangles to estimate distances.

#### CPCTC
CPCTC is an abbreviation for the phrase “Corresponding Parts of Congruent Triangles are Congruent.” It can be used as a justification in a proof after you have proven two triangles congruent.

### Example 1
**Engineering Application**
To design a bridge across a canyon, you need to find the distance from A to B. Locate points C, D, and E as shown in the figure. If $DE = 600\text{ ft}$, what is $AB$?

- $\angle D \cong \angle B$, because they are both right angles.
- $DC \cong CB$, because $DC = CB = 500\text{ ft}$.
- $\angle DCE \cong \angle BCA$, because vertical angles are congruent. Therefore $\triangle DCE \cong \triangle BCA$ by ASA or LA. By CPCTC, $ED \cong AB$, so $AB = ED = 600\text{ ft}$.

### Example 2
**Proving Corresponding Parts Congruent**

Given: $AB \cong DC$, $\angle ABC \cong \angle DCB$

Prove: $\angle A \cong \angle D$

**Proof:**

1. $AB \cong DC$  
   Given

2. $\angle ABC \cong \angle DCB$  
   Given

3. $\triangle ABC \cong \triangle DCB$  
   SAS

4. $\angle A \cong \angle D$  
   CPCTC

**Check it Out!**

1. A landscape architect sets up the triangles shown in the figure to find the distance $JK$ across a pond. What is $JK$?

2. Given: $\overline{PR}$ bisects $\angle QPS$ and $\angle QRS$.  
   Prove: $\overline{PQ} \cong \overline{PS}$
**Example 3**

**Using CPCTC in a Proof**

Given: \(EG \parallel DF, EG \cong DF\)
Prove: \(ED \parallel GF\)

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (EG \cong DF)</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. (EG \parallel DF)</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. (\angle EGD \cong \angle FDG)</td>
<td>3. Alt. Int. (\triangle) Thm.</td>
</tr>
<tr>
<td>4. (GD \cong DG)</td>
<td>4. Reflex. Prop. of (\cong)</td>
</tr>
<tr>
<td>5. (\triangle EGD \cong \triangle FDG)</td>
<td>5. SAS <em>Steps 1, 3, and 4</em></td>
</tr>
<tr>
<td>6. (\angle EDG \cong \angle FGD)</td>
<td>6. CPCTC</td>
</tr>
<tr>
<td>7. (ED \parallel GF)</td>
<td>7. Converse of Alt. Int. (\triangle) Thm.</td>
</tr>
</tbody>
</table>

**Check it out!**

3. Given: \(J\) is the midpoint of \(KL\) and \(NL\).
   Prove: \(KL \parallel MN\)

You can also use CPCTC when triangles are on a coordinate plane.
You use the Distance Formula to find the lengths of the sides of each triangle.
Then, after showing that the triangles are congruent, you can make conclusions about their corresponding parts.

**Example 4**

**Using CPCTC in the Coordinate Plane**

Given: \(A(2, 3), B(5, -1), C(1, 0), D(-4, -1), E(0, 2), F(-1, -2)\)
Prove: \(\angle ABC \cong \angle DEF\)

**Step 1** Plot the points on a coordinate plane.

**Step 2** Use the Distance Formula to find the lengths of the sides of each triangle.

\[
D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
AB = \sqrt{(5 - 2)^2 + (-1 - 3)^2} = \sqrt{9 + 16} = \sqrt{25} = 5
\]

\[
DE = \sqrt{(0 - (-4))^2 + (2 - (-1))^2} = \sqrt{16 + 9} = \sqrt{25} = 5
\]

\[
BC = \sqrt{(1 - 5)^2 + (0 - (-1))^2} = \sqrt{16 + 1} = \sqrt{17}
\]

\[
EF = \sqrt{(-1 - 0)^2 + (-2 - 2)^2} = \sqrt{1 + 16} = \sqrt{17}
\]

\[
AC = \sqrt{(1 - 2)^2 + (0 - 3)^2} = \sqrt{1 + 9} = \sqrt{10}
\]

\[
DF = \sqrt{(-1 - (-4))^2 + (-2 - (-1))^2} = \sqrt{9 + 1} = \sqrt{10}
\]

So \(AB \cong DE, BC \cong EF, \) and \(AC \cong DF\). Therefore \(\triangle ABC \cong \triangle DEF\) by SSS, and \(\angle ABC \cong \angle DEF\) by CPCTC.

**Check it out!**

4. Given: \(J(-1, -2), K(2, -1), L(-2, 0), R(2, 3), S(5, 2), T(1, 1)\)
   Prove: \(\angle JKL \cong \angle RST\)
**Guided Practice**

1. **Vocabulary** You use CPCTC after proving triangles are congruent. Which parts of congruent triangles are referred to as corresponding parts?

2. **Engineering** To find the height of a windmill, a rancher places a marker at C and steps off the distance from C to B. Then the rancher walks the same distance from C in the opposite direction and places a marker at D. If DE = 6.3 m, what is AB?

3. **Given:** X is the midpoint of ST. RX ⊥ ST
   **Prove:** RS ≅ RT
   **Proof:**
   - RX ⊥ ST
   - Given
   - ∠RXS and ∠RXT are rt. ∆.
   - a. ?
   - RX ≅ RX
   - b. ?
   - X is the mdpt. of ST.
   - c. ?
   - SX ≅ TX
   - d. ?
   - e. ?
   - SAS
   - f. ?
   - RS ≅ RT
4. Given: \( \overline{AC} \cong \overline{AD}, \overline{CB} \cong \overline{DB} \)
   Prove: \( \overline{AB} \) bisects \( \angle CAD \).

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{AC} \cong \overline{AD}, \overline{CB} \cong \overline{DB} )</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. b. ?</td>
<td>2. Reflex. Prop. of ( \cong )</td>
</tr>
<tr>
<td>3. ( \triangle ACB \cong \triangle ADB )</td>
<td>3. c. ?</td>
</tr>
<tr>
<td>4. ( \angle CAB \cong \angle DAB )</td>
<td>4. d. ?</td>
</tr>
<tr>
<td>5. ( \overline{AB} ) bisects ( \angle CAD )</td>
<td>5. e. ?</td>
</tr>
</tbody>
</table>

Multi-Step  Use the given set of points to prove each congruence statement.

5. \( E(-3, 3), F(-1, 3), G(-2, 0), J(0, -1), K(2, -1), L(1, 2); \overline{EFG} \cong \overline{JKL} \)

6. \( A(2, 3), B(4, 1), C(1, -1), R(-1, 0), S(-3, -2), T(0, -4); \overline{ACB} \cong \overline{RTS} \)

PRACTICE AND PROBLEM SOLVING

7. **Surverying** To find the distance \( AB \) across a river, a surveyor first locates point \( C \).
   He measures the distance from \( C \) to \( B \). Then he locates point \( D \) the same distance east of \( C \). If \( DE = 420 \text{ ft} \), what is \( AB \)?

8. Given: \( M \) is the midpoint of \( PQ \) and \( RS \).
   Prove: \( \overline{QR} \cong \overline{PS} \)

9. Given: \( WX \cong XY \cong YZ \cong ZW \)
   Prove: \( \angle W \cong \angle Y \)

10. Given: \( G \) is the midpoint of \( \overline{FH} \).
    \( EF \cong EH \)
    Prove: \( \angle 1 \cong \angle 2 \)

11. Given: \( \overline{LM} \) bisects \( \angle JKL \).
    \( JL \cong KL \)
    Prove: \( M \) is the midpoint of \( JK \).

Multi-Step  Use the given set of points to prove each congruence statement.

12. \( R(0, 0), S(2, 4), T(-1, 3), U(-1, 0), V(-3, -4), W(-4, -1); \overline{RST} \cong \overline{UVW} \)

13. \( A(-1, 1), B(2, 3), C(2, -2), D(2, -3), E(-1, -5), F(-1, 0); \overline{BAC} \cong \overline{EDF} \)

14. Given: \( \triangle QRS \) is adjacent to \( \triangle QTS \).
    \( \overline{QS} \) bisects \( \angle RQT \).
    \( \angle R \cong \angle T \)
    Prove: \( QS \) bisects \( R\overline{T} \).

15. Given: \( \triangle ABE \) and \( \triangle CDE \) with \( E \) the midpoint of \( \overline{AC} \) and \( \overline{BD} \)
    Prove: \( \overline{AB} \parallel \overline{CD} \)
16. This problem will prepare you for the Multi-Step TAKS Prep on page 280.
The front of a doghouse has the dimensions shown.
   a. How can you prove that \( \triangle ADB \cong \triangle ADC \)?
   b. Prove that \( BD \cong CD \).
   c. What is the length of \( BD \) and \( BC \) to the nearest tenth?

Multi-Step  Find the value of \( x \).
17. \[
x + 11 = 2x - 3
\]
18. \[
\begin{align*}
(4x + 1)^\circ & = (6x - 41)^\circ \\
\end{align*}
\]
Use the diagram for Exercises 19–21.
19. Given: \( PS = RQ \), \( m \angle 1 = m \angle 4 \)
   Prove: \( m \angle 3 = m \angle 2 \)
20. Given: \( m \angle 1 = m \angle 2 \), \( m \angle 3 = m \angle 4 \)
   Prove: \( PS = RS \)
21. Given: \( PS = RQ \), \( PQ = RS \)
   Prove: \( \overline{PQ} \parallel \overline{RS} \)
22. Critical Thinking Does the diagram contain enough information to allow you to conclude that \( JK \parallel ML \)? Explain.
23. Write About It  Draw a diagram and explain how a surveyor can set up triangles to find the distance across a lake. Label each part of your diagram. List which sides or angles must be congruent.

24. Which of these will NOT be used as a reason in a proof of \( \triangle AC \cong \triangle AD \)?
   A) SAS  C) ASA
   B) CPCTC  D) Reflexive Property

25. Given the points \( K(1, 2), L(0, -4), M(-2, -3), \) and \( N(-1, 3) \), which of these is true?
   F) \( \angle KNL \cong \angle MNL \)
   H) \( \angle MLN \cong \angle KLN \)
   G) \( \angle LNK \cong \angle NLM \)
   J) \( \angle MNK \cong \angle NKL \)

26. What is the value of \( y \)?
   \[
   (10x + y)^2 = 40x + \frac{5}{2}
   \]
   A) 10  C) 35
   B) 20  D) 85

27. Which of these are NOT used to prove angles congruent?
   F) congruent triangles  H) parallel lines
   G) noncorresponding parts  I) perpendicular lines
28. Which set of coordinates represents the vertices of a triangle congruent to \( \triangle RST \)? (Hint: Find the lengths of the sides of \( \triangle RST \).)

\[ \text{A: } (3, 4), (3, 0), (0, 0) \quad \text{C: } (3, 1), (3, 3), (4, 6) \]
\[ \text{B: } (3, 3), (0, 4), (0, 0) \quad \text{D: } (3, 0), (4, 4), (0, 6) \]

**CHALLENGE AND EXTEND**

29. All of the edges of a cube are congruent. All of the angles on each face of a cube are right angles. Use CPCTC to explain why any two diagonals on the faces of a cube (for example, \( AC \) and \( AF \)) must be congruent.

30. Given: \( JK \cong ML, JM \cong KL \)
Prove: \( \angle J \cong \angle L \)
(Hint: Draw an auxiliary line.)

31. Given: \( S \) is the midpoint of \( DC \).
Prove: \( \triangle ASD \cong \triangle BSC \)

32. \( \triangle ABC \) is in plane \( \mathcal{M} \). \( \triangle CDE \) is in plane \( \mathcal{P} \). Both planes have \( C \) in common and \( \angle A \cong \angle E \). What is the height \( AB \) to the nearest foot?

**SPIRAL REVIEW**

33. Lina's test scores in her history class are 90, 84, 93, 88, and 91. What is the minimum score Lina must make on her next test to have an average test score of 90? (Previous course)

34. One long-distance phone plan costs $3.95 per month plus $0.08 per minute of use. A second long-distance plan costs $0.10 per minute for the first 50 minutes used each month and then $0.15 per minute after that. Which plan is cheaper if you use an average of 75 long-distance minutes per month? (Previous course)

A figure has vertices at \((1, 3), (2, 2), (3, 2), \) and \((4, 3)\). Identify the transformation of the figure that produces an image with each set of vertices. (Lesson 1-7)

35. \((1, -3), (2, -2), (3, -2), (4, -3)\)
36. \((-2, -1), (-1, -2), (0, -2), (1, -1)\)
37. Determine if you can use ASA to prove \( \triangle ACB \cong \triangle ECD \). Explain. (Lesson 4-5)
Quadratic Equations

Algebra

A quadratic equation is an equation that can be written in the form $ax^2 + bx + c = 0$.

See Skills Bank page S66

Example

Given: $\triangle ABC$ is isosceles with $\overline{AB} \cong \overline{AC}$. Solve for $x$.

Step 1 Set $x^2 - 5x$ equal to 6 to get $x^2 - 5x = 6$.

Step 2 Rewrite the quadratic equation by subtracting 6 from each side to get $x^2 - 5x - 6 = 0$.

Step 3 Solve for $x$.

Method 1: Factoring

$x^2 - 5x - 6 = 0$

$(x - 6)(x + 1) = 0$

$x - 6 = 0$ or $x + 1 = 0$

$x = 6$ or $x = -1$

Method 2: Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-6)}}{2(1)}$

$x = \frac{5 \pm \sqrt{49}}{2}$

$x = \frac{5 \pm 7}{2}$

$x = \frac{12}{2}$ or $x = \frac{-2}{2}$

$x = 6$ or $x = -1$

Step 4 Check each solution in the original equation.

$x^2 - 5x = 6$

$(6)^2 - 5(6) = 6$

$36 - 30 = 6$

$6 - 6 \checkmark$

$x^2 - 5x = 6$

$(-1)^2 - 5(-1) = 6$

$1 + 5 = 6$

$6 - 6 \checkmark$

Try This

TAKS Grades 9–11 Obj. 5, 6

Solve for $x$ in each isosceles triangle.

1. Given: $\overline{FE} \cong \overline{FG}$

2. Given: $\overline{JK} \cong \overline{JL}$

3. Given: $\overline{XY} \cong \overline{YZ}$

4. Given: $\overline{QP} \cong \overline{QR}$

Chapter 4 Triangle Congruence
Who uses this?
The Bushmen in South Africa use the Global Positioning System to transmit data about endangered animals to conservationists. (See Exercise 24.)

You have used coordinate geometry to find the midpoint of a line segment and to find the distance between two points. Coordinate geometry can also be used to prove conjectures.

A coordinate proof is a style of proof that uses coordinate geometry and algebra. The first step of a coordinate proof is to position the given figure in the plane. You can use any position, but some strategies can make the steps of the proof simpler.

### Strategies for Positioning Figures in the Coordinate Plane

- Use the origin as a vertex, keeping the figure in Quadrant I.
- Center the figure at the origin.
- Center a side of the figure at the origin.
- Use one or both axes as sides of the figure.

### Example 1

**Positioning a Figure in the Coordinate Plane**

Position a rectangle with a length of 8 units and a width of 3 units in the coordinate plane.

**Method 1** You can center the longer side of the rectangle at the origin.

**Method 2** You can use the origin as a vertex of the rectangle.

Depending on what you are using the figure to prove, one solution may be better than the other. For example, if you need to find the midpoint of the longer side, use the first solution.

1. Position a right triangle with leg lengths of 2 and 4 units in the coordinate plane. (Hint: Use the origin as the vertex of the right angle.)
Once the figure is placed in the coordinate plane, you can use slope, the coordinates of the vertices, the Distance Formula, or the Midpoint Formula to prove statements about the figure.

**Example 2**

Writing a Proof Using Coordinate Geometry

Write a coordinate proof.

Given: Right \( \triangle ABC \) has vertices \( A(0, 6) \), \( B(0, 0) \), and \( C(4, 0) \). \( D \) is the midpoint of \( \overline{AC} \).

Prove: The area of \( \triangle DBC \) is one half the area of \( \triangle ABC \).

Proof: \( \triangle ABC \) is a right triangle with height \( AB \) and base \( BC \).

\[
\text{area of } \triangle ABC = \frac{1}{2}bh = \frac{1}{2}(4)(6) = 12 \text{ square units}
\]

By the Midpoint Formula, the coordinates of \( D \) are \( \left(\frac{0 + 4}{2}, \frac{6 + 0}{2}\right) = (2, 3) \). The \( y \)-coordinate of \( D \) is the height of \( \triangle DBC \), and the base is 4 units.

\[
\text{area of } \triangle DBC = \frac{1}{2}bh = \frac{1}{2}(4)(3) = 6 \text{ square units}
\]

Since \( 6 = \frac{1}{2}(12) \), the area of \( \triangle DBC \) is one half the area of \( \triangle ABC \).

**Check It Out!**

2. Use the information in Example 2 to write a coordinate proof showing that the area of \( \triangle ADB \) is one half the area of \( \triangle ABC \).

A coordinate proof can also be used to prove that a certain relationship is always true. You can prove that a statement is true for all right triangles without knowing the side lengths. To do this, assign variables as the coordinates of the vertices.

**Example 3**

Assigning Coordinates to Vertices

Position each figure in the coordinate plane and give the coordinates of each vertex.

**A** a right triangle with leg lengths \( a \) and \( b \)

\[ (0, 0) \quad (0, a) \quad (b, 0) \]

**B** a rectangle with length \( c \) and width \( d \)

\[ (0, 0) \quad (0, d) \quad (c, 0) \quad (c, d) \]

3. Position a square with side length \( 4p \) in the coordinate plane and give the coordinates of each vertex.

If a coordinate proof requires calculations with fractions, choose coordinates that make the calculations simpler. For example, use multiples of 2 when you are to find coordinates of a midpoint. Once you have assigned the coordinates of the vertices, the procedure for the proof is the same, except that your calculations will involve variables.
Writing a Coordinate Proof

Given: \( \angle B \) is a right angle in \( \triangle ABC \). \( D \) is the midpoint of \( \overline{AC} \).
Prove: The area of \( \triangle DBC \) is one half the area of \( \triangle ABC \).

Step 1 Assign coordinates to each vertex.

The coordinates of \( A \) are \((0, 2j)\),
the coordinates of \( B \) are \((0, 0)\),
and the coordinates of \( C \) are \((2n, 0)\).

Step 2 Position the figure in the coordinate plane.

Step 3 Write a coordinate proof.

Proof: \( \triangle ABC \) is a right triangle with height \( 2j \) and base \( 2n \).

\[
\text{area of } \triangle ABC = \frac{1}{2}bh \\
= \frac{1}{2}(2n)(2j) \\
= 2nj \text{ square units}
\]

By the Midpoint Formula, the coordinates of \( D = \left( \frac{0 + 2n}{2}, \frac{2j + 0}{2} \right) = (n, j) \).

The height of \( \triangle DBC \) is \( j \) units, and the base is \( 2n \) units.

\[
\text{area of } \triangle DBC = \frac{1}{2}bh \\
= \frac{1}{2}(2n)(j) \\
= nj \text{ square units}
\]

Since \( nj = \frac{1}{2}(2nj) \), the area of \( \triangle DBC \) is one half the area of \( \triangle ABC \).

4. Use the information in Example 4 to write a coordinate proof showing that the area of \( \triangle ADB \) is one half the area of \( \triangle ABC \).

THINK AND DISCUSS

1. When writing a coordinate proof why are variables used instead of numbers as coordinates for the vertices of a figure?

2. How does the way you position a figure in the coordinate plane affect your calculations in a coordinate proof?

3. Explain why it might be useful to assign \( 2p \) as a coordinate instead of just \( p \).

4. GET ORGANIZED Copy and complete the graphic organizer.

In each row, draw an example of each strategy that might be used when positioning a figure for a coordinate proof.

<table>
<thead>
<tr>
<th>Positioning Strategy</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use origin as a vertex.</td>
<td></td>
</tr>
<tr>
<td>Center figure at origin.</td>
<td></td>
</tr>
<tr>
<td>Center side of figure at origin.</td>
<td></td>
</tr>
<tr>
<td>Use axes as sides of figure.</td>
<td></td>
</tr>
</tbody>
</table>
GUIDED PRACTICE

1. **Vocabulary** What is the relationship between coordinate geometry, coordinate plane, and coordinate proof?

Position each figure in the coordinate plane.

2. a rectangle with a length of 4 units and width of 1 unit
3. a right triangle with leg lengths of 1 unit and 3 units

Write a proof using coordinate geometry.

4. Given: Right \( \triangle PQR \) has coordinates \( P(0, 6) \), \( Q(8, 0) \), and \( R(0, 0) \). \( A \) is the midpoint of \( PR \).
   \( B \) is the midpoint of \( QR \).
   Prove: \( AB = \frac{1}{2}PQ \)

Position each figure in the coordinate plane and give the coordinates of each vertex.

5. a right triangle with leg lengths \( m \) and \( n \)
6. a rectangle with length \( a \) and width \( b \)

Multi-Step Assign coordinates to each vertex and write a coordinate proof.

7. Given: \( \angle R \) is a right angle in \( \triangle PQR \). \( A \) is the midpoint of \( PR \).
   \( B \) is the midpoint of \( QR \).
   Prove: \( AB = \frac{1}{2}PQ \)

PRACTICE AND PROBLEM SOLVING

Position each figure in the coordinate plane.

8. a square with side lengths of 2 units
9. a right triangle with leg lengths of 1 unit and 5 units

Write a proof using coordinate geometry.

10. Given: Rectangle \( ABCD \) has coordinates \( A(0, 0) \), \( B(0, 10) \), \( C(6, 10) \), and \( D(6, 0) \). \( E \) is the midpoint of \( AB \), and \( F \) is the midpoint of \( CD \).
    Prove: \( EF = BC \)

Position each figure in the coordinate plane and give the coordinates of each vertex.

11. a square with side length \( 2m \)
12. a rectangle with dimensions \( x \) and \( 3x \)

Multi-Step Assign coordinates to each vertex and write a coordinate proof.

13. Given: \( E \) is the midpoint of \( AB \) in rectangle \( ABCD \). \( F \) is the midpoint of \( CD \).
    Prove: \( EF = AD \)

14. Critical Thinking Use variables to write the general form of the endpoints of a segment whose midpoint is \( (0, 0) \).
15. **Recreation** A hiking trail begins at $E(0, 0)$. Bryan hikes from the start of the trail to a waterfall at $W(3, 3)$ and then makes a $90^\circ$ turn to a campsite at $C(6, 0)$.
   a. Draw Bryan's route in the coordinate plane.
   b. If one grid unit represents 1 mile, what is the total distance Bryan hiked? Round to the nearest tenth.

Find the perimeter and area of each figure.
16. a right triangle with leg lengths of $a$ and $2a$ units
17. a rectangle with dimensions $s$ and $t$ units

Find the missing coordinates for each figure.
18. \[ \begin{align*}
(0, n) & \quad (m, m) \\
(0, 0) & \quad (n, 0)
\end{align*} \]
19. \[ \begin{align*}
(0, 0) & \quad (m, n) \\
(0, 0) & \quad (n, n)
\end{align*} \]

20. **Conservation** The Bushmen have sighted animals at the following coordinates: $(-25, 31.5), (-23.2, 31.4),$ and $(-24, 31.1)$. Prove that the distance between two of these locations is approximately twice the distance between two other.

21. **Navigation** Two ships depart from a port at $P(20, 10)$. The first ship travels to a location at $A(-30, 50)$, and the second ship travels to a location at $B(70, -30)$. Each unit represents one nautical mile. Find the distance to the nearest nautical mile between the two ships. Verify that the port is at the midpoint between the two.

Write a coordinate proof.
22. **Given:** Rectangle $PQRS$ has coordinates $P(0, 2), Q(3, 2), R(3, 0),$ and $S(0, 0)$. $PR$ and $QS$ intersect at $T(1.5, 1)$.
   **Prove:** The area of $\triangle RST$ is $\frac{1}{4}$ of the area of the rectangle.
23. **Given:** $A(x_1, y_1), B(x_2, y_2)$, with midpoint $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
   **Prove:** $AM = \frac{1}{2}AB$
24. Plot the points on a coordinate plane and connect them to form $\triangle KLM$ and $\triangle MPK$. Write a coordinate proof.
   **Given:** $K(-2, 1), L(-2, 3), M(1, 3), P(1, 1)$
   **Prove:** $\triangle KLM \cong \triangle MPK$

25. **Write About It** When you place two sides of a figure on the coordinate axes, what are you assuming about the figure?

26. This problem will prepare you for the Multi-Step TAKS Prep on page 280.
   Paul designed a doghouse to fit against the side of his house. His plan consisted of a right triangle on top of a rectangle.
   a. Find $BD$ and $CE$.
   b. Before building the doghouse, Paul sketched his plan on a coordinate plane. He placed $A$ at the origin and $AB$ on the $x$-axis. Find the coordinates of $B, C, D,$ and $E$, assuming that each unit of the coordinate plane represents one inch.
27. The coordinates of the vertices of a right triangle are (0, 0), (4, 0), and (0, 2). Which is a true statement?
   A) The vertex of the right angle is at (4, 2).
   B) The midpoints of the two legs are at (2, 0) and (0, 1).
   C) The hypotenuse of the triangle is $\sqrt{6}$ units.
   D) The shortest side of the triangle is positioned on the x-axis.

28. A rectangle has dimensions of 2g and 2f units. If one vertex is at the origin, which coordinates could NOT represent another vertex?
   F) (2f, g)  G) (2f, 0)  H) (2g, 2f)  J) (−2f, 2g)

29. The coordinates of the vertices of a rectangle are (0, 0), (a, 0), (a, b), and (0, b). What is the perimeter of the rectangle?
   A) $a + b$  B) $ab$  C) $\frac{1}{2}ab$  D) $2a + 2b$

30. A coordinate grid is placed over a map. City A is located at (−1, 2) and city C is located at (3, 5). If city C is at the midpoint between city A and city B, what are the coordinates of city B?
   F) (1, 3.5)  G) (−5, −1)  H) (7, 8)  J) (2, 7)

CHALLENGE AND EXTEND
Find the missing coordinates for each figure.

31. 
   \[
   \begin{align*}
   &y \quad x \\
   &(a, 0) \quad (a + c, 0) \\
   &(a, b) \quad (\_\_, \_\_) \\
   
   \end{align*}
   \]

32. 
   \[
   \begin{align*}
   &y \quad x \\
   &\left(n, h\right) \quad \left(n + p, h\right) \\
   &(0, 0) \quad (\_\_, \_\_) \\
   
   \end{align*}
   \]

33. The vertices of a right triangle are at (−2s, 2s), (0, 2s), and (0, 0). What coordinates could be used so that a coordinate proof would be easier to complete?

34. Rectangle ABCD has dimensions of 2f and 2g units. The equation of the line containing $BD$ is $y = \frac{g}{f}x$, and the equation of the line containing $AC$ is $y = -\frac{g}{f}x + 2g$. Use algebra to show that the coordinates of E are $(f, g)$.

SPIRAL REVIEW
Use the quadratic formula to solve for $x$. Round to the nearest hundredth if necessary. (Previous course)

35. $0 = 8x^2 + 18x - 5$  36. $0 = x^2 + 3x - 5$  37. $0 = 3x^2 - x - 10$

Find each value. (Lesson 3-2)

38. $x$

39. $y$

40. Use $A(-4, 3), B(-1, 3), C(-3, 1), D(0, -2)$, $E(3, -2)$, and $F(2, -4)$ to prove $\angle ABC \cong \angle EDF$. (Lesson 4-6).
4-8 Isosceles and Equilateral Triangles

Objectives
Prove theorems about isosceles and equilateral triangles.
Apply properties of isosceles and equilateral triangles.

Vocabulary
legs of an isosceles triangle
vertex angle
base
base angles

Who uses this?
Astronomers use geometric methods. (See Example 1.)

Recall that an isosceles triangle has at least two congruent sides. The congruent sides are called the legs. The vertex angle is the angle formed by the legs. The side opposite the vertex angle is called the base, and the base angles are the two angles that have the base as a side.

\[ \angle 3 \text{ is the vertex angle.} \]
\[ \angle 1 \text{ and } \angle 2 \text{ are the base angles.} \]

### Theorems

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-8-1</td>
<td>Isosceles Triangle Theorem</td>
<td>( \triangle ABC )</td>
</tr>
<tr>
<td></td>
<td>If two sides of a triangle are congruent, then the angles opposite the sides are congruent.</td>
<td>( \angle B \cong \angle C )</td>
</tr>
<tr>
<td>4-8-2</td>
<td>Converse of Isosceles Triangle Theorem</td>
<td>( \triangle DEF )</td>
</tr>
<tr>
<td></td>
<td>If two angles of a triangle are congruent, then the sides opposite those angles are congruent.</td>
<td>( DE \cong DF )</td>
</tr>
</tbody>
</table>

Theorem 4-8-1 is proven below. You will prove Theorem 4-8-2 in Exercise 35.

### Proof

**Isosceles Triangle Theorem**

*Given:* \( AB \cong AC \)

*Prove:* \( \angle B \cong \angle C \)

*Proof:*

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Draw ( X ), the mdpt. of ( BC ).</td>
<td>1. Every seg. has a unique mdpt.</td>
</tr>
<tr>
<td>2. Draw the auxiliary line ( AX ).</td>
<td>2. Through two pts. there is exactly one line.</td>
</tr>
<tr>
<td>3. ( BX \cong CX )</td>
<td>3. Def. of mdpt.</td>
</tr>
<tr>
<td>4. ( AB \cong AC )</td>
<td>4. Given</td>
</tr>
<tr>
<td>5. ( AX \cong AX )</td>
<td>5. Reflex. Prop. of ( \cong )</td>
</tr>
<tr>
<td>6. ( \triangle ABX \cong \triangle ACX )</td>
<td>6. SSS Steps 3, 4, 5</td>
</tr>
<tr>
<td>7. ( \angle B \cong \angle C )</td>
<td>7. CPCTC</td>
</tr>
</tbody>
</table>
EXAMPLE 1  
Astronomy Application

The distance from Earth to nearby stars can be measured using the parallax method, which requires observing the positions of a star 6 months apart. If the distance $LM$ to a star in July is $4.0 \times 10^{13}$ km, explain why the distance $LK$ to the star in January is the same. (Assume the distance from Earth to the Sun does not change.)

$m\angle LKM = 180 - 90.4$, so $m\angle LKM = 89.6^\circ$. Since $\angle LKM \cong \angle M$, $\triangle LMK$ is isosceles by the Converse of the Isosceles Triangle Theorem. Thus $LK = LM = 4.0 \times 10^{13}$ km.

1. If the distance from Earth to a star in September is $4.2 \times 10^{13}$ km, what is the distance from Earth to the star in March? Explain.

EXAMPLE 2  
Finding the Measure of an Angle

Find each angle measure.

A $m\angle C$

$m\angle C = m\angle B = x^\circ$

$m\angle C + m\angle B + m\angle A = 180$

$x + x + 38 = 180$

$2x = 142$

$x = 71$

Thus $m\angle C = 71^\circ$.

B $m\angle S$

$m\angle S = m\angle R$

$2x^\circ = (x + 30)^\circ$

$x = 30$

Thus $m\angle S = 2x^\circ = 2(30) = 60^\circ$.

The following corollary and its converse show the connection between equilateral triangles and equiangular triangles.

Corollary 4-8-3  
Equilateral Triangle

<table>
<thead>
<tr>
<th>COROLLARY</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>If a triangle is equilateral, then it is equiangular.</td>
<td>$\triangle ABC$</td>
<td>$\angle A = \angle B = \angle C$</td>
</tr>
</tbody>
</table>

You will prove Corollary 4-8-3 in Exercise 36.
**Corollary 4-8-4  Equiangular Triangle**

<table>
<thead>
<tr>
<th>COROLLARY</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>If a triangle is equiangular, then it is equilateral.((\text{equiangular } \triangle \rightarrow \text{equilateral } \triangle))</td>
<td>(\triangle DEF \equiv \triangle DEF \equiv \triangle EF)</td>
<td></td>
</tr>
</tbody>
</table>

You will prove Corollary 4-8-4 in Exercise 37.

**Example 3  Using Properties of Equilateral Triangles**

Find each value.

**A**
- \(x\)
- \(\triangle ABC\) is equiangular.
- \((3x + 15)^\circ = 60^\circ\)
- \(3x = 45\)
- \(x = 15\)

**B**
- \(t\)
- \(\triangle JKL\) is equilateral.
- \(4t - 8 = 2t + 1\)
- \(2t = 9\)
- \(t = 4.5\)

**Check It Out!**

3. Use the diagram to find \(JL\).

**Example 4  Using Coordinate Proof**

Prove that the triangle whose vertices are the midpoints of the sides of an isosceles triangle is also isosceles.

**Given:** \(\triangle ABC\) is isosceles. \(X\) is the mdpt. of \(AB\). \(Y\) is the mdpt. of \(AC\). \(Z\) is the mdpt. of \(BC\).

**Prove:** \(\triangle XYZ\) is isosceles.

**Proof:**

Draw a diagram and place the coordinates of \(\triangle ABC\) and \(\triangle XYZ\) as shown.

By the Midpoint Formula, the coordinates of \(X\) are \((\frac{2a + 0}{2}, \frac{2b + 0}{2}) = (a, b)\), the coordinates of \(Y\) are \((\frac{2a + 4a}{2}, \frac{2b + 0}{2}) = (3a, b)\), and the coordinates of \(Z\) are \((\frac{4a + 0}{2}, \frac{0 + 0}{2}) = (2a, 0)\).

By the Distance Formula, \(XZ = \sqrt{(2a - a)^2 + (0 - b)^2} = \sqrt{a^2 + b^2}\), and \(YZ = \sqrt{(2a - 3a)^2 + (0 - b)^2} = \sqrt{a^2 + b^2}\).

Since \(XZ = YZ\), \(\overline{XZ} \cong \overline{YZ}\) by definition. So \(\triangle XYZ\) is isosceles.

**Check It Out!**

4. **What if...?** The coordinates of \(\triangle ABC\) are \(A(0, 2b)\), \(B(-2a, 0)\), and \(C(2a, 0)\). Prove \(\triangle XYZ\) is isosceles.
THINK AND DISCUSS

1. Explain why each of the angles in an equilateral triangle measures 60°.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, draw and mark a diagram for each type of triangle.

GUIDED PRACTICE

1. Vocabulary Draw isosceles \( \triangle JKL \) with \( \angle K \) as the vertex angle. Name the legs, base, and base angles of the triangle.

2. Surveying To find the distance \( QR \) across a river, a surveyor locates three points \( Q, R, \) and \( S \). \( QS = 41 \) m, and \( m\angle S = 35° \). The measure of exterior \( \angle PQS = 70° \). Draw a diagram and explain how you can find \( QR \).

Find each angle measure.

3. \( m\angle ECD \)

4. \( m\angle K \)

5. \( m\angle X \)

6. \( m\angle A \)

Find each value.

7. \( y \)

8. \( x \)

9. \( BC \)

10. \( JK \)

11. Given: \( \triangle ABC \) is right isosceles. \( X \) is the midpoint of \( AC \). \( AB \equiv BC \)

Prove: \( \triangle AXB \) is isosceles.
12. **Aviation** A plane is flying parallel to the ground along $\overrightarrow{AC}$. When the plane is at $A$, an air-traffic controller in tower $T$ measures the angle to the plane as 40°. After the plane has traveled 2.4 mi to $B$, the angle to the plane is 80°. How can you find $BT$?

Find each angle measure.

13. $m\angle E$

14. $m\angle TRU$

15. $m\angle F$

16. $m\angle A$

Find each value.

17. $z$

18. $y$

19. $BC$

20. $XZ$

21. Given: $\triangle ABC$ is isosceles. $P$ is the midpoint of $AB$. $Q$ is the midpoint of $AC$.
   $AB \cong AC$
   Prove: $PC \cong QB$

Tell whether each statement is sometimes, always, or never true.
Support your answer with a sketch.

22. An equilateral triangle is an isosceles triangle.

23. The vertex angle of an isosceles triangle is congruent to the base angles.

24. An isosceles triangle is a right triangle.

25. An equilateral triangle and an obtuse triangle are congruent.

26. **Critical Thinking** Can a base angle of an isosceles triangle be an obtuse angle? Why or why not?
27. This problem will prepare you for the Multi-Step TAKS Prep on page 280.
The diagram shows the inside view of the support structure of the back of a doghouse. \( \overline{PQ} \cong \overline{PR} \), \( \overline{PS} \cong \overline{PT} \), \( m\angle PST = 71^\circ \), and \( m\angle QPS = m\angle RPT = 18^\circ \).

a. Find \( m\angle SPT \).
b. Find \( m\angle PQR \) and \( m\angle PRQ \).

Multi-Step Find the measure of each numbered angle.

28. \( 2 \) \( 1 \) \( 3 \) \( 58^\circ \)

29. \( 3 \) \( 1 \) \( 2 \) \( 74^\circ \)

30. Write a coordinate proof.
   
   Given: \( \angle B \) is a right angle in isosceles right \( \triangle ABC \).
   
   \( X \) is the midpoint of \( \overline{AC} \), \( \overline{BA} \equiv \overline{BC} \)
   
   Prove: \( \triangle AXB \cong \triangle CXB \)

31. Estimation Draw the figure formed by \((-2, 1), (5, 5), \) and \((-1, -7)\). Estimate the measure of each angle and make a conjecture about the classification of the figure. Then use a protractor to measure each angle. Was your conjecture correct? Why or why not?

32. How many different isosceles triangles have a perimeter of 18 and sides whose lengths are natural numbers? Explain.

Multi-Step Find the value of the variable in each diagram.

33.

34.

35. Prove the Converse of the Isosceles Triangle Theorem.

36. Complete the proof of Corollary 4-8-3.

Given: \( \overline{AB} \equiv \overline{AC} \equiv \overline{BC} \)

Prove: \( \angle A \equiv \angle B \equiv \angle C \)

Proof: Since \( \overline{AB} \equiv \overline{AC} \), a. \(?\) by the Isosceles Triangle Theorem.

Since \( \overline{AC} \equiv \overline{BC} \), \( \angle A \equiv \angle B \) by b. \(?\). Therefore \( \angle A \equiv \angle C \) by c. \(?\).

By the Transitive Property of \( \equiv \), \( \angle A \equiv \angle B \equiv \angle C \).

37. Prove Corollary 4-8-4.

38. Navigation The captain of a ship traveling along \( \overline{AB} \) sights an island \( C \) at an angle of 45°. The captain measures the distance the ship covers until it reaches \( B \), where the angle to the island is 90°. Explain how to find the distance \( BC \) to the island.

39. Given: \( \triangle ABC \cong \triangle CBA \)

Prove: \( \triangle ABC \) is isosceles.

40. Write About It Write the Isosceles Triangle Theorem and its converse as a biconditional.
41. Rewrite the paragraph proof of the Hypotenuse-Leg (HL) Congruence Theorem as a two-column proof.

**Given:** △ABC and △DEF are right triangles. ∠C and ∠F are right angles. \( \overline{AC} \cong \overline{DF} \) and \( \overline{AB} \cong \overline{DE} \).

**Prove:** △ABC \( \cong \) △DEF

**Proof:** On △DEF draw \( \overline{EF} \). Mark G so that \( \overline{FG} \cong \overline{CB} \). Thus \( \overline{FG} \cong \overline{DB} \). From the diagram, \( \overline{AC} \cong \overline{DF} \) and \( \angle C \) and \( \angle F \) are right angles. \( \overline{DF} \perp \overline{EG} \) by definition of perpendicular lines. Thus \( \angle DFG \) is a right angle, and \( \angle DFG \cong \angle C \). △ABC \( \cong \) △DGF by SAS. \( \overline{DG} \cong \overline{AB} \) by CPCTC. \( \overline{AB} \cong \overline{DE} \) as given. \( \overline{DG} \cong \overline{DE} \) by the Transitive Property. By the Isosceles Triangle Theorem \( \angle G \cong \angle E \). \( \angle DFG \cong \angle DFE \) since right angles are congruent. So △DGF \( \cong \) △DEF by AAS. Therefore △ABC \( \cong \) △DEF by the Transitive Property.

42. Lorena is designing a window so that \( \angle R \), \( \angle S \), \( \angle T \), and \( \angle U \) are right angles, \( \overline{VU} \cong \overline{VT} \), and \( m \angle UVT = 20^\circ \).

What is \( m \angle RUV \)?

- A 10°
- B 70°
- C 20°
- D 80°

43. Which of these values of \( y \) makes △ABC isosceles?

- F \( \frac{1}{4} + \frac{1}{2} \)
- G \( 2 \frac{1}{2} \)
- H \( 7 \frac{1}{2} \)
- I \( 15 \frac{1}{2} \)

44. **Gridded Response** The vertex angle of an isosceles triangle measures \( (6t - 9)^\circ \), and one of the base angles measures \( (4t)^\circ \). Find \( t \).

**CHALLENGE AND EXTEND**

45. In the figure, \( \overline{JK} \cong \overline{IL} \) and \( \overline{KM} \cong \overline{KL} \). Let \( m \angle J = x^\circ \). Prove \( m \angle MKL \) must also be \( x^\circ \).

46. An equilateral △ABC is placed on a coordinate plane. Each side length measures \( 2a \). B is at the origin, and C is at \((2a, 0)\). Find the coordinates of A.

47. An isosceles triangle has coordinates \( A(0, 0) \) and \( B(a, b) \). What are all possible coordinates of the third vertex?

**SPIRAL REVIEW**

Find the solutions for each equation. *(Previous course)*

48. \( x^2 + 5x + 4 = 0 \)

49. \( x^2 - 4x + 3 = 0 \)

50. \( x^2 - 2x + 1 = 0 \)

Find the slope of the line that passes through each pair of points. *(Lesson 3-5)*

51. \((2, -1)\) and \((0, 5)\)

52. \((-5, -10)\) and \((20, -10)\)

53. \((4, 7)\) and \((10, 11)\)

54. Position a square with a perimeter of 4s in the coordinate plane and give the coordinates of each vertex. *(Lesson 4-7)*
Proving Triangles Congruent

Gone to the Dogs  You are planning to build a doghouse for your dog. The pitched roof of the doghouse will be supported by four trusses. Each truss will be an isosceles triangle with the dimensions shown. To determine the materials you need to purchase and how you will construct the trusses, you must first plan carefully.

1. You want to be sure that all four trusses are exactly the same size and shape. Explain how you could measure three lengths on each truss to ensure this. Which postulate or theorem are you using?

2. Prove that the two triangular halves of the truss are congruent.

3. What can you say about \( \overline{AD} \) and \( \overline{DB} \)? Why is this true? Use this to help you find the lengths of \( \overline{AD}, \overline{DB}, \overline{AC}, \) and \( \overline{BC} \).

4. You want to make careful plans on a coordinate plane before you begin your construction of the trusses. Each unit of the coordinate plane represents 1 inch. How could you assign coordinates to vertices \( A, B, \) and \( C \)?

5. \( m\angle ACB = 106^\circ \). What is the measure of each of the acute angles in the truss? Explain how you found your answer.

6. You can buy the wood for the trusses at the building supply store for $0.80 a foot. The store sells the wood in 6-foot lengths only. How much will you have to spend to get enough wood for the 4 trusses of the doghouse?
Quiz for Lessons 4-4 Through 4-8

4-4 Triangle Congruence: SSS and SAS

1. The figure shows one tower and the cables of a suspension bridge. Given that \( AC \cong BC \), use SAS to explain why \( \triangle ACD \cong \triangle BCD \).

2. Given: \( JK \) bisects \( \angle MJN \). \( MJ \cong NJ \)

   Prove: \( \triangle MJK \cong \triangle NJK \)

4-5 Triangle Congruence: ASA, AAS, and HL

Determine if you can use the HL Congruence Theorem to prove the triangles congruent. If not, tell what else you need to know.

3. \( \triangle RSU \) and \( \triangle TUS \)

4. \( \triangle ABC \) and \( \triangle DBC \)

Observers in two lighthouses \( K \) and \( L \) spot a ship \( S \).

5. Draw a diagram of the triangle formed by the lighthouses and the ship. Label each measure.

6. Is there enough data in the table to pinpoint the location of the ship? Why?

4-6 Triangle Congruence: CPCTC

7. Given: \( CD \parallel BE, DE \parallel CB \)

   Prove: \( \angle D \cong \angle B \)

4-7 Introduction to Coordinate Proof

8. Position a square with side lengths of 9 units in the coordinate plane

9. Assign coordinates to each vertex and write a coordinate proof.

   Given: \( ABCD \) is a rectangle with \( M \) as the midpoint of \( AB \). \( N \) is the midpoint of \( AD \).

   Prove: The area of \( \triangle AMN \) is \( \frac{1}{8} \) the area of rectangle \( ABCD \).

4-8 Isosceles and Equilateral Triangles

Find each value.

10. \( m \angle C \)

11. \( ST \)

12. Given: Isosceles \( \triangle JKL \) has coordinates \( J(0, 0), K(2a, 2b) \), and \( L(4a, 0) \).

   \( M \) is the midpoint of \( JK \), and \( N \) is the midpoint of \( KL \).

   Prove: \( \triangle KMN \) is isosceles.
When performing a compass and straightedge construction, the compass setting remains the same width until you change it. This fact allows you to construct a segment congruent to a given segment. You can assume that two distances constructed with the same compass setting are congruent.

The steps in the construction of a figure can be justified by combining the assumptions of compass and straightedge constructions and the postulates and theorems that are used for proving triangles congruent.

You have learned that there exists exactly one midpoint on any line segment. The proof below justifies the construction of a midpoint.

**EXAMPLE 1**

**Proving the Construction of a Midpoint**

**Given:** diagram showing the steps in the construction

**Prove:** $M$ is the midpoint of $\overline{AB}$.

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Draw $\overline{AC}$, $\overline{BC}$, $\overline{AD}$, and $\overline{BD}$.</td>
<td>1. Through any two pts. there is exactly one line.</td>
</tr>
<tr>
<td>2. $\overline{AC} \cong \overline{BC} \cong \overline{AD} \cong \overline{BD}$</td>
<td>2. Same compass setting used</td>
</tr>
<tr>
<td>3. $\overline{CD} \cong \overline{CD}$</td>
<td>3. Reflex. Prop. of $\cong$</td>
</tr>
<tr>
<td>4. $\triangle ACD \cong \triangle BCD$</td>
<td>4. SSS <em>Steps 2, 3</em></td>
</tr>
<tr>
<td>5. $\angle ACD \cong \angle BCD$</td>
<td>5. CPCTC</td>
</tr>
<tr>
<td>6. $\overline{CM} \cong \overline{CM}$</td>
<td>6. Reflex. Prop. of $\cong$</td>
</tr>
<tr>
<td>7. $\triangle ACM \cong \triangle BCM$</td>
<td>7. SAS <em>Steps 2, 5, 6</em></td>
</tr>
<tr>
<td>8. $\overline{AM} \cong \overline{BM}$</td>
<td>8. CPCTC</td>
</tr>
<tr>
<td>9. $M$ is the midpt. of $\overline{AB}$</td>
<td>9. Def. of mdpt.</td>
</tr>
</tbody>
</table>

1. Given: above diagram

Prove: $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$.
Example 2: Proving the Construction of an Angle

Given: diagram showing the steps in the construction

Prove: \( \angle A \cong \angle D \)

Proof: Since there is a straight line through any two points, you can draw \( \overline{BC} \) and \( \overline{EF} \). The same compass setting was used to construct \( \overline{AC}, \overline{AB}, \overline{DF}, \) and \( \overline{DE} \), so \( AC \cong AB \cong DF \cong DE \). The same compass setting was used to construct \( \overline{BC} \) and \( \overline{EF} \), so \( BC \cong EF \). Therefore \( \triangle BAC \cong \triangle EDF \) by SSS, and \( \angle A \cong \angle D \) by CPCTC.

Check it out! 2. Prove the construction for bisecting an angle. (See page 23.)

Extension

Exercises

Use each diagram to prove the construction valid.

1. parallel lines
   (See page 163 and page 170.)

2. a perpendicular through a point not on the line (See page 179.)

3. constructing a triangle using SAS
   (See page 243.)

4. constructing a triangle using ASA
   (See page 253.)
**Vocabulary**

acute triangle .................. 216  
auxiliary line ................... 223  
base .............................. 273  
base angle ....................... 273  
congruent polygons .......... 231  
coordinate proof ............. 267  
corollary ........................ 224  
corresponding angles ...... 231  
corresponding sides .......... 231  
CPCTC .......................... 260  
isosceles triangle .......... 217  
legs of an isosceles triangle .. 273  
equiangular triangle ........ 216  
equilateral triangle .......... 217  
obtuse triangle ................ 216  
remote interior angle ....... 225  
right triangle ................... 216  
scalene triangle ............... 217  
triangle rigidity ............. 242  
vertex angle ................... 273

Complete the sentences below with vocabulary words from the list above.

1. A(n) **is a triangle with at least two congruent sides.**

2. A name given to matching angles of congruent triangles is **.**

3. A(n) **is the common side of two consecutive angles in a polygon.**

---

**4-1 Classifying Triangles (pp. 216–221)**

**Example**

Classify the triangle by its angle measures and side lengths.

- **isosceles right triangle**

**Exercises**

Classify each triangle by its angle measures and side lengths.

4. 

5. 

---

**4-2 Angle Relationships in Triangles (pp. 223–230)**

**Example**

- **Find \(m \angle S\).**

- **Find \(m \angle N\).**

6. 

7. In \(\triangle LMN\), \(m \angle L = 8x^\circ\), \(m \angle M = (2x + 1)^\circ\), and \(m \angle N = (6x - 1)^\circ\).
**4-3 Congruent Triangles (pp. 231–237)**

**EXAMPLE**

- Given: \( \triangle DEF \cong \triangle JKL \). Identify all pairs of congruent corresponding parts. Then find the value of \( x \).

The congruent pairs follow: \( \angle D \cong \angle J \), \( \angle E \cong \angle K \), \( \angle F \cong \angle L \), \( \overline{DE} \cong \overline{JK} \), \( \overline{EF} \cong \overline{KL} \), and \( \overline{DF} \cong \overline{JL} \).

Since \( \angle E = \angle K \), \( 90 = 8x - 22 \). After 22 is added to both sides, \( 112 = 8x \). So \( x = 14 \).

**EXERCISES**

8. \( \overline{PR} \cong \overline{?} \)

9. \( \angle Y \cong \overline{?} \)

**4-4 Triangle Congruence: SSS and SAS (pp. 242–249)**

**EXAMPLES**

- Given: \( \overline{RS} \cong \overline{UT} \), and \( \overline{VS} \cong \overline{VT} \). \( V \) is the midpoint of \( \overline{RU} \).

Prove: \( \triangle RSV \cong \triangle UTV \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{RS} \cong \overline{UT} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{VS} \cong \overline{VT} )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( V ) is the mdpt. of ( \overline{RU} )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \overline{RV} \cong \overline{UV} )</td>
<td>4. Def. of mdpt.</td>
</tr>
<tr>
<td>5. ( \triangle RSV \cong \triangle UTV )</td>
<td>5. SSS Steps 1, 2, 4</td>
</tr>
</tbody>
</table>

- Show that \( \triangle ADB \cong \triangle CDB \) when \( s = 5 \).

\[
\begin{align*}
AB &= s^2 - 4s \\
AD &= 14 - 2s \\
&= 5^2 - 4(5) \\
&= 14 - 2(5) \\
&= 5 \\
&= 4 \\
\overline{BD} &\cong \overline{BD} \text{ by the Reflexive Property.} \\
\overline{AD} &\cong \overline{CD} \\
\overline{AB} &\cong \overline{CB} \text{ So } \triangle ADB \cong \triangle CDB \text{ by SSS.}
\end{align*}
\]

**EXERCISES**

12. Given: \( \overline{AB} \cong \overline{DE} \), \( \overline{DB} \cong \overline{AE} \).

Prove: \( \triangle ADB \cong \triangle ADE \)

13. Given: \( \overline{GJ} \) bisects \( \overline{FH} \), and \( \overline{FH} \) bisects \( \overline{GJ} \).

Prove: \( \triangle FGK \cong \triangle HJK \)

14. Show that \( \triangle ABC \cong \triangle XYZ \) when \( x = -6 \).

15. Show that \( \triangle LMN \cong \triangle PQR \) when \( y = 25 \).
### Chapter 4 Triangle Congruence: ASA, AAS, and HL (pp. 252–259)

#### 4-5 Triangle Congruence: ASA, AAS, and HL

**Examples**

- Given: \( B \) is the midpoint of \( \overline{AE} \).
  \[ \angle A \cong \angle E, \quad \angle ABC \cong \angle EBD \]
  Prove: \( \triangle ABC \cong \triangle EBD \)

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle A \cong \angle E )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle ABC \cong \angle EBD )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( B ) is the mdpt. of ( \overline{AE} )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \overline{AB} \cong \overline{EB} )</td>
<td>4. Def. of mdpt.</td>
</tr>
<tr>
<td>5. ( \triangle ABC \cong \triangle EBD )</td>
<td>5. ASA Steps 1, 4, 2</td>
</tr>
</tbody>
</table>

**Exercises**

16. Given: \( C \) is the midpoint of \( \overline{AG} \).
  \( HA \parallel GB \)
  Prove: \( \triangle HAC \cong \triangle BGC \)

17. Given: \( \overline{WX} \perp \overline{XZ} \),
  \( \overline{YZ} \perp \overline{ZX} \),
  \( \overline{WZ} \cong \overline{YX} \)
  Prove: \( \triangle WZX \cong \triangle YXZ \)

18. Given: \( \angle S \) and \( \angle V \) are right angles.
  \( RT = UW \),
  \( m\angle T = m\angle W \)
  Prove: \( \triangle RST \cong \triangle UVW \)

### Chapter 4 Triangle Congruence: CPCTC (pp. 260–265)

#### 4-6 Triangle Congruence: CPCTC

**Examples**

- Given: \( \overline{JL} \) and \( \overline{HK} \) bisect each other.
  Prove: \( \angle JHG \cong \angle LKG \)

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{JL} ) and ( \overline{HK} ) bisect each other.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{JG} \cong \overline{LG} ), and ( \overline{HG} \cong \overline{KG} ).</td>
<td>2. Def. of bisect</td>
</tr>
<tr>
<td>3. ( \angle JGH \cong \angle LGK )</td>
<td>3. Vert. ( \triangle ) Thm.</td>
</tr>
<tr>
<td>4. ( \triangle JHG \cong \triangle LKG )</td>
<td>4. SAS Steps 2, 3</td>
</tr>
<tr>
<td>5. ( \angle JHG \cong \angle LKG )</td>
<td>5. CPCTC</td>
</tr>
</tbody>
</table>

**Exercises**

19. Given: \( M \) is the midpoint of \( \overline{BD} \).
  \( \overline{BC} \cong \overline{DC} \)
  Prove: \( \angle 1 \cong \angle 2 \)

20. Given: \( \overline{PQ} \cong \overline{RQ} \),
  \( \overline{PS} \cong \overline{RS} \)
  Prove: \( QS \) bisects \( \angle PQR \).

21. Given: \( H \) is the midpoint of \( \overline{GJ} \).
  \( L \) is the midpoint of \( \overline{MK} \).
  \( \overline{GM} \cong \overline{KJ} \), \( \overline{GJ} \cong \overline{KM} \), \( \angle G \cong \angle K \)
  Prove: \( \angle GMH \cong \angle KJL \)
### 4-7 Introduction to Coordinate Proof (pp. 267–272)

**EXAMPLES**

- Given: $\angle B$ is a right angle in isosceles right $\triangle ABC$. $E$ is the midpoint of $\overline{AB}$. $D$ is the midpoint of $\overline{CB}$. $\overline{AB} \cong \overline{CB}$

  Prove: $\overline{CE} \cong \overline{AD}$

  **Proof:** Use the coordinates $A(0, 2a)$, $B(0, 0)$, and $C(2a, 0)$. Draw $\overline{AD}$ and $\overline{CE}$.

  By the Midpoint Formula,
  
  $E = \left( \frac{0 + 0}{2}, \frac{2a + 0}{2} \right) = (0, a)$ and $D = \left( \frac{0 + 2a}{2}, \frac{0 + 0}{2} \right) = (a, 0)$

  By the Distance Formula,
  
  $CE = \sqrt{(2a - 0)^2 + (0 - a)^2} = \sqrt{4a^2 + a^2} = a\sqrt{5}$
  
  $AD = \sqrt{(a - 0)^2 + (0 - 2a)^2} = \sqrt{a^2 + 4a^2} = a\sqrt{5}$

  Thus $\overline{CE} \cong \overline{AD}$ by the definition of congruence.

### 4-8 Isosceles and Equilateral Triangles (pp. 273–279)

**EXAMPLE**

- Find the value of $x$.

  $m\angle D + m\angle E + m\angle F = 180^\circ$

  by the Triangle Sum Theorem.

  $m\angle E = m\angle F$

  by the Isosceles Triangle Theorem.

  $m\angle D + 2m\angle E = 180^\circ$

  $42 + 2(3x) = 180$

  $6x = 138$

  $x = 23$

### EXERCISES

Position each figure in the coordinate plane and give the coordinates of each vertex.

22. a right triangle with leg lengths $r$ and $s$

23. a rectangle with length $2p$ and width $p$

24. a square with side length $8m$

For exercises 25 and 26 assign coordinates to each vertex and write a coordinate proof.

25. Given: In rectangle $ABCD$, $E$ is the midpoint of $\overline{AB}$, $F$ is the midpoint of $\overline{BC}$, $G$ is the midpoint of $\overline{CD}$, and $H$ is the midpoint of $\overline{AD}$.

  Prove: $\overline{EF} \cong GH$

26. Given: $\triangle PQR$ has a right $\angle Q$.

  $M$ is the midpoint of $\overline{PR}$.

  Prove: $MP = MQ = MR$

27. Show that a triangle with vertices at $(3, 5)$, $(3, 2)$, and $(2, 5)$ is a right triangle.

28. Find the value of $x$.

29. $RS$

30. Given: $\triangle ACD$ is isosceles with $\angle D$ as the vertex angle. $B$ is the midpoint of $\overline{AC}$.

  $AB = x + 5$, $BC = 2x - 3$, and $CD = 2x + 6$.

  Find the perimeter of $\triangle ACD$. 

### Study Guide: Review (pp. 267–272)

1. Classify \( \triangle ACD \) by its angle measures.

Classify each triangle by its side lengths.
2. \( \triangle ACD \)
3. \( \triangle ABC \)
4. \( \triangle ABD \)

5. While surveying the triangular plot of land shown, a surveyor finds that \( m\angle S = 43^\circ \). The measure of \( \angle RTP \) is twice that of \( \angle RTS \). What is \( m\angle R \)?

Given: \( \triangle XYZ \cong \triangle JKL \)
Identify the congruent corresponding parts.
6. \( \overline{JL} \cong ? \)
7. \( \angle Y \cong ? \)
8. \( \angle L \cong ? \)
9. \( \overline{YZ} \cong ? \)

10. Given: \( T \) is the midpoint of \( PR \) and \( SQ \).
    Prove: \( \triangle PTS \cong \triangle RTQ \)

11. The figure represents a walkway with triangular supports. Given that \( \overline{GJ} \) bisects \( \angle HGK \) and \( \angle H \cong \angle K \), use AAS to prove \( \triangle HGJ \cong \triangle KGJ \)

12. Given: \( \overline{AB} \cong \overline{DC} \),
    \( \overline{AB} \perp \overline{AC} \),
    \( \overline{DC} \perp \overline{DB} \)
    Prove: \( \triangle ABC \cong \triangle DCB \)

13. Given: \( \overline{PQ} \parallel \overline{SR} \),
    \( \angle S \cong \angle Q \)
    Prove: \( \overline{PS} \parallel \overline{QR} \)

14. Position a right triangle with legs 3 m and 4 m long in the coordinate plane. Give the coordinates of each vertex.

15. Assign coordinates to each vertex and write a coordinate proof.
    Given: \( \text{Square } ABCD \)
    Prove: \( \overline{AC} \cong \overline{BD} \)

Find each value.
16. \( y \)

17. \( m\angle S \)

18. Given: Isosceles \( \triangle ABC \) has coordinates \( A(2a, 0) \), \( B(0, 2b) \), and \( C(-2a, 0) \).
    \( D \) is the midpoint of \( \overline{AC} \), and \( E \) is the midpoint of \( \overline{AB} \).
    Prove: \( \triangle AED \) is isosceles.
FOCUS ON ACT

The ACT Mathematics Test is one of four tests in the ACT. You are given 60 minutes to answer 60 multiple-choice questions. The questions cover material typically taught through the end of eleventh grade. You will need to know basic formulas but nothing too difficult.

You may want to time yourself as you take this practice test. It should take you about 5 minutes to complete.

1. For the figure below, which of the following must be true?

![Diagram](image)

I. \( \angle EFG > \angle DEF \)
II. \( \angle EDF = \angle EFD \)
III. \( \angle DEF + \angle EDF > \angle EFG \)

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

2. In the figure below, \( \triangle ABD \cong \triangle CDB \), \( \angle A = (2x + 14)° \), \( \angle C = (3x - 15)° \), and \( \angle DBA = 49° \). What is the measure of \( \angle BDA \)?

![Diagram](image)

(F) 29°
(G) 49°
(H) 59°
(J) 72°
(K) 101°

3. Which of the following best describes a triangle with vertices having coordinates \((-1, 0), (0, 3), \) and \((1, -4)\)?

(A) Equilateral
(B) Isosceles
(C) Right
(D) Scalene
(E) Equiangular

4. In the figure below, what is the value of \( y \)?

![Diagram](image)

(F) 49
(G) 87
(H) 93
(J) 131
(K) 136

5. In \( \triangle RST \), \( RS = 2x + 10 \), \( ST = 3x - 2 \), and \( RT = \frac{1}{2}x + 28 \). If \( \triangle RST \) is equiangular, what is the value of \( x \)?

(A) 2
(B) \( 5 \frac{1}{3} \)
(C) 6
(D) 12
(E) 34
Any Question Type: Identify Key Words and Context Clues

When reading a test item, you should pay attention to key words and context clues given in the problem statement. These clues will guide you in providing a correct response.

**Example 1**

**Multiple Choice** What is the side length of an equilateral triangle with a perimeter of $42\frac{3}{4}$ cm?

- **A** $42\frac{3}{4}$ cm
- **B** $24\frac{3}{4}$ cm
- **C** $21\frac{3}{8}$ cm
- **D** $14\frac{1}{4}$ cm

**LOOK** for key words and context clues and underline them. Identify what they mean.

What is the side length of an **equilateral triangle** with a **perimeter** of $42\frac{3}{4}$ in.?

- **equilateral triangle** → a triangle with three congruent sides
- **perimeter** → the distance around a figure
- **perimeter** = 3 (length of one side)

You find the perimeter of an equilateral triangle by multiplying the length of one side of the triangle by three.

\[
\frac{42\frac{3}{4}}{3} = 3(x)
\]

The correct choice is **D** because the length of the side of the equilateral triangle is $14\frac{1}{4}$ cm.

**Example 2**

**Gridded Response** The vertex angle of an isosceles triangle measures $(5t - 5)^\circ$, and one of the base angles measures $(t + 5)^\circ$. Find $t$.

- **isosceles triangle** → a triangle with at least two congruent sides
- **vertex angle** → the angle formed by the legs
- **base angles** → The side opposite the vertex angle is called the base, and the base angles are the two angles that have the base as a side.

\[
2(\text{measure of the base angle}) + \text{(measure of the vertex angle)} = 180^\circ
\]

\[
2(t + 5) + (5t - 5) = 180
\]

\[
2t + 10 + 5t - 5 = 180
\]

\[
7t + 5 = 180
\]

\[
t = 25
\]

The correct value for $t$ is 25.
If you do not understand what a word means, reread the sentences that contain the word and make a logical guess.

Read each test item and answer the questions that follow.

**Item A**
**Multiple Choice** Which value of \( k \) would make \( \triangle CDE \) isosceles?

![Diagram of \( \triangle CDE \) with vertices labeled \( C \), \( D \), and \( E \), and sides labeled \( 2k + 1 \), \( 5k - 6 \), \( 3k - 7 \), \( 3k - 7 \), \( 5k - 6 \), and \( 2k + 1 \).]

- A) 7
- B) 6
- C) \( \frac{2}{3} \)
- D) \( \frac{1}{3} \)

1. Whether a triangle is isosceles depends on what characteristics of the triangle?
2. What do \( 2k + 1 \), \( 3k - 7 \), and \( 5k - 6 \) represent in the model?
3. How will you use the definition of an isosceles triangle to find the correct value of \( k \)?

**Item B**
**Gridded Response** What must the value of \( x \) be in order to prove that \( \triangle MNQ \cong \triangle PNQ \) by SSS?

![Diagram of \( \triangle MNQ \) with vertices labeled \( M \), \( N \), and \( Q \), and sides labeled \( 2x + 5 \), \( 8x - 7 \), \( 12 \), \( 12 \), and \( 2x + 5 \).]

4. What statement are you trying to prove?
5. Explain the meaning of the symbol \( \cong \).

6. How will you use the abbreviation SSS to help you answer the question?

**Item C**
**Multiple Choice** \( \angle X \) and \( \angle Y \) are the remote interior angles of \( \angle YZW \) in \( \triangle XYZ \). Which of these equations must be true?

- F) \( 180^\circ - m\angle X = m\angle YZW \)
- G) \( m\angle X = m\angle Y + 90^\circ \)
- H) \( m\angle X = m\angle YZW - m\angle Y \)
- J) \( m\angle YZW = m\angle YZX - m\angle YXZ \)

7. Create a drawing that represents the situation. Label the remote interior angles.
8. What is the relationship between the remote interior angles and an exterior angle?
9. How can you manipulate the relationship given in Problem 8 to get one of the four choices?

**Item D**
**Multiple Choice** Which of the following is a correct classification of \( \triangle FGH \)?

![Diagram of \( \triangle FGH \) with sides labeled 4 in., 45 in., and 90 in.]

- A) Acute
- B) Equiangular
- C) Isosceles
- D) Scalene

10. What are the two ways by which triangles can be classified?
11. What must be true for the triangle to be classified as acute? as equiangular?
12. What must be true for the triangle to be classified as isosceles? as scalene?
CUMULATIVE ASSESSMENT, CHAPTERS 1–4

Multiple Choice

Use the diagram for Items 1 and 2.

1. Which of these congruence statements can be proved from the information given in the figure?
   - A $\triangle AEB \cong \triangle CED$
   - B $\triangle BAC \cong \triangle DAC$
   - C $\triangle ABD \cong \triangle BCA$
   - D $\triangle DEC \cong \triangle DEA$

2. What other information is needed to prove that $\triangle CEB \cong \triangle AED$ by the HL Congruence Theorem?
   - A $AD \cong AB$
   - B $CB \cong AD$
   - C $BE \cong AE$
   - D $DE \cong CE$

3. Which biconditional statement is true?
   - A Tomorrow is Monday if and only if today is not Saturday.
   - B Next month is January if and only if this month is December.
   - C Today is a weekend day if and only if yesterday was Friday.
   - D This month had 31 days if and only if last month had 30 days.

4. What must be true if $\overrightarrow{PQ}$ intersects $\overrightarrow{ST}$ at more than one point?
   - A $P, Q, S, and T$ are collinear.
   - B $P, Q, S, and T$ are noncoplanar.
   - C $\overrightarrow{PQ}$ and $\overrightarrow{ST}$ are opposite rays.
   - D $\overrightarrow{PQ}$ and $\overrightarrow{ST}$ are perpendicular.

5. $\triangle ABC \cong \triangle DEF$, $EF = x^2 - 7$, and $BC = 4x - 2$. Find the values of $x$.
   - A $-1$ and $5$
   - B $-1$ and $6$
   - C $1$ and $5$
   - D $2$ and $3$

6. Which conditional statement has the same truth value as its inverse?
   - F If $n < 0$, then $n^2 > 0$.
   - G If a triangle has three congruent sides, then it is an isosceles triangle.
   - H If an angle measures less than $90^\circ$, then it is an acute angle.
   - J If $n$ is a negative integer, then $n < 0$.

7. On a map, an island has coordinates $(3, 5)$, and a reef has coordinates $(6, 8)$. If each map unit represents 1 mile, what is the distance between the island and the reef to the nearest tenth of a mile?
   - A 4.2 miles
   - B 6.0 miles
   - C 9.0 miles
   - D 15.8 miles

8. A line has an $x$-intercept of $-8$ and a $y$-intercept of $3$. What is the equation of the line?
   - A $y = -8x + 3$
   - B $y = \frac{3}{8}x + 3$
   - C $y = \frac{8}{3}x - 8$
   - D $y = 3x - 8$

9. $\overrightarrow{JK}$ passes through points $J(1, 3)$ and $K(-3, 11)$. Which of these lines is perpendicular to $\overrightarrow{JK}$?
   - A $y = -\frac{1}{2}x + \frac{1}{3}$
   - B $y = \frac{1}{2}x + 6$
   - C $y = -2x - \frac{1}{5}$
   - D $y = 2x - 4$

10. If $PQ = 2(RS) + 4$ and $RS = TU + 1$, which equation is true by the Substitution Property of Equality?
    - F $PQ = TU + 5$
    - G $PQ = TU + 6$
    - H $PQ = 2(TU) + 5$
    - J $PQ = 2(TU) + 6$

11. Which of the following is NOT valid for proving that triangles are congruent?
    - A AAA
    - B ASA
    - C SAS
    - D HL
12. What is the measure of \( \angle ACD \)?
- F 40°
- G 80°
- H 100°
- J 140°

13. What type of triangle is \( \triangle ABC \)?
- A Isosceles acute
- B Equilateral acute
- C Isosceles obtuse
- D Scalene acute

14. \( \triangle CDE \cong \triangle JKL \). \( m \angle E = (3x + 4)^\circ \), and \( m \angle L = (6x - 5)^\circ \). What is the value of \( x \)?

15. Lucy, Eduardo, Carmen, and Frank live on the same street. Eduardo’s house is halfway between Lucy’s house and Frank’s house. Lucy’s house is halfway between Carmen’s house and Frank’s house. If the distance between Eduardo’s house and Lucy’s house is 150 ft, what is the distance in feet between Carmen’s house and Eduardo’s house?

16. \( \triangle JKL \cong \triangle XYZ \), and \( JK = 10 - 2n \). \( XY = 2 \), and \( YZ = n \). Find \( KL \).

17. An angle is its own supplement. What is its measure?

18. The area of a circle is 154 square inches. What is its circumference to the nearest inch?

19. The measure of \( \angle P \) is \( 3\frac{1}{2} \) times the measure of \( \angle Q \). If \( \angle P \) and \( \angle Q \) are complementary, what is \( m \angle P \) in degrees?

20. Given \( \ell \parallel m \) with transversal \( n \), explain why \( \angle 2 \) and \( \angle 3 \) are complementary.

21. \( \angle G \) and \( \angle H \) are supplementary angles. \( m \angle G = (2x + 12)^\circ \), and \( m \angle H = x^\circ \).
   a. Write an equation that can be used to determine the value of \( x \). Solve the equation and justify each step.
   b. Explain why \( \angle H \) has a complement but \( \angle G \) does not.

22. A manager conjectures that for every 1000 parts a factory produces, 60 are defective.
   a. If the factory produces 1500 parts in one day, how many of them can be expected to be defective based on the manager’s conjecture? Explain how you found your answer.
   b. Use the data in the table below to show that the manager’s conjecture is false.

<table>
<thead>
<tr>
<th>Day</th>
<th>Parts</th>
<th>Defective Parts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
<td>150</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>1500</td>
<td>90</td>
</tr>
<tr>
<td>5</td>
<td>2500</td>
<td>150</td>
</tr>
</tbody>
</table>

23. \( BD \) is the perpendicular bisector of \( \overline{AC} \).
   a. What are the conclusions you can make from this statement?
   b. Suppose \( BD \) intersects \( \overline{AC} \) at \( D \). Explain why \( BD \) is the shortest path from \( B \) to \( \overline{AC} \).

24. \( \triangle ABC \) and \( \triangle DEF \) are isosceles triangles. \( \overline{BC} \cong \overline{EF} \), and \( \overline{AC} \cong \overline{DF} \). \( m \angle C = 42.5^\circ \), and \( m \angle E = 95^\circ \).
   a. What is \( m \angle D \)? Explain how you determined your answer.
   b. Show that \( \triangle ABC \) and \( \triangle DEF \) are congruent.
   c. Given that \( EF = 2x + 7 \) and \( AB = 3x + 2 \), find the value for \( x \). Explain how you determined your answer.
Cavanaugh Flight Museum

Located on the grounds of the Addison Airport in Addison, Texas, the Cavanaugh Flight Museum offers visitors a thrilling voyage through the history of U.S. military flight. The museum exhibits aircraft and other military artifacts on almost 50,000 square feet of display area. It also features an informative self-guided tour and offers rides in two classic airplanes.

Choose one or more strategies to solve each problem.

1. The table gives data on some of the aircraft on display in the museum. Suppose the number of each type of aircraft declines at the same rate each year. Find the yearly rate of change for each given year. Round to the nearest whole number.

<table>
<thead>
<tr>
<th>Airplane</th>
<th>Number Built</th>
<th>Number Left in 2005</th>
<th>Year First Built</th>
</tr>
</thead>
<tbody>
<tr>
<td>N2S-4 Stearman</td>
<td>10,346</td>
<td>2136</td>
<td>1933</td>
</tr>
<tr>
<td>Fairchild PT-19</td>
<td>4,889</td>
<td>272</td>
<td>1938</td>
</tr>
<tr>
<td>Spitfire Mk VIII</td>
<td>20,334</td>
<td>70</td>
<td>1943</td>
</tr>
<tr>
<td>F9F-2 Panther</td>
<td>761</td>
<td>9</td>
<td>1947</td>
</tr>
</tbody>
</table>

2. Visitors to the museum can see a replica of an N2S-4 Stearman "Yellow Peril," a plane used to train pilots during World War II. The plane has two parallel wings $AB$ and $CD$ that are connected by bracing wires. The wires are arranged such that $\angle EFG = 29^\circ$ and $GF$ bisects $\angle EGD$. What is $\angle AEG$?

3. Visitors have the opportunity to ride in either the N2S-4 Stearman or the AT-6 Texan. If the airport uses two cameras mounted 1000 ft apart to determine the position of a plane during landing and takeoff, what is the distance $d$ that the plane in the diagram has moved along the runway since it passed camera 1?
**The Great Texas Balloon Race**

The annual Great Texas Balloon Race is one of the most exciting hot air balloon events in Texas. “Balloon Glow,” in which balloons are tethered and illuminated in an evening display, was begun in Longview, the race’s starting point, in 1980. Traditionally held in July, the race attracts balloonists who compete to fly the obstacle course the most accurately.

Choose one or more strategies to solve each problem.

1. The event starts in Longview, and ends near Estes, Texas. The balloons do not fly from the start to the finish in a straight line. They follow a zigzag course to take advantage of the wind. Suppose one of the balloons leaves Longview at a bearing of N 50° E and follows the course shown. At what bearing does the balloon approach Estes?

   ![Diagram of Longview to Estes course]

   **Bearing Distance (mi)**
   
<table>
<thead>
<tr>
<th>L to X</th>
<th>N 42° E</th>
<th>3.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>X to Y</td>
<td>S 59° E</td>
<td>2.4</td>
</tr>
<tr>
<td>Y to L</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. The speed of the balloon depends on the current wind speed. One event in The Great Texas Balloon Race requires the balloonist to fly to a pole that is 2 mi from the starting point. The balloonist must drop a small ring around the pole, which is 20 ft tall. A second target is 1 mi from the first, a third target is another 3 mi from the second, and a final target is 5 mi farther. If the wind speed is 3.5 mi/h, how long will it take the balloonist to finish the course? Round to the nearest hundredth of an hour.

3. During the race, one of the balloons leaves Longview \( L \), flies to \( X \), and then flies to \( Y \). The team discovers a problem with the balloon, so it must return directly to Longview. Does the table contain enough information to determine the return course to \( L \)? Explain.